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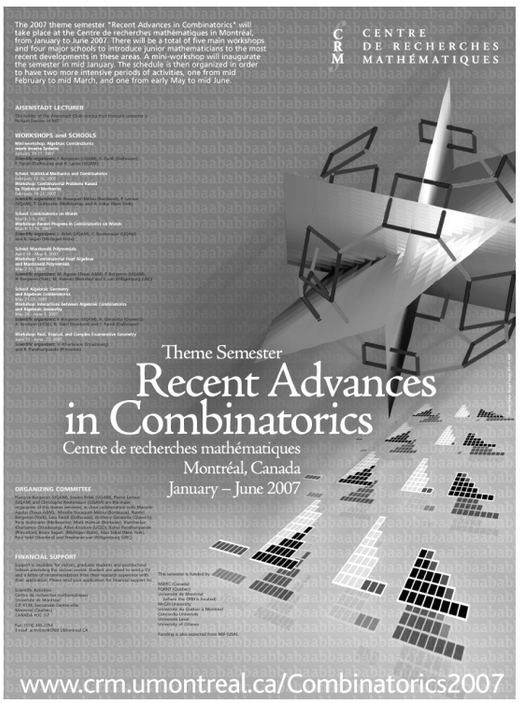
Two Theme Semesters around Combinatorics for 2006 – 2007

THE core of each year's scientific program at the CRM is its thematic program. Until now, the whole academic year was organized around a single theme. Hencefoth, it will be subdivided into two thematic semesters, the emphasis being on pure mathematics during one of them and applied mathematics during the other. Accordingly, the first semester of the 2006 – 2007 academic year will be devoted to Combinatorial Optimization, while the second one will deal with Recent Advances in Combinatorics.

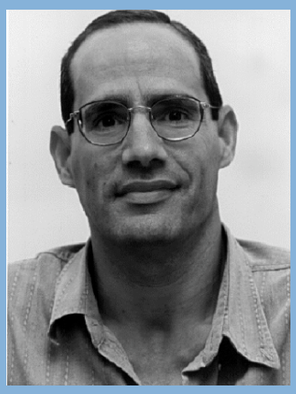
The organizing committee for the Combinatorial Optimization semester comprises David Avis (McGill), David Bremner (New Brunswick), Vašek Chvátal (Concordia), Bill Cunningham (Waterloo), Michel Goemans (MIT), Pierre Hansen (HÉC Montréal), Odile Marcotte (UQÀM), and Adrian Vetta (McGill). The André-Aisenstadt Chair will be held by Noga Alon (Tel Aviv) and Paul Seymour (Princeton), who will give series of lectures at the CRM during the fall of 2006.

The main organizers of the program on Recent Advances in Combinatorics are François Bergeron (UQÀM), Srecko Brlek (UQÀM), Pierre Leroux (UQÀM), and Christophe Reutenauer (UQÀM). The holder of the André-Aisenstadt Chair during this thematic semester will be Richard Stanley of MIT. The semester will end with a workshop organised by Slava Kharlamov (Strasbourg) and Rahul Pandharipande (Princeton) on enumerative problems in complex, real and tropical geometries.

More information on these two programs can be found on the next two pages.



The 2006 – 2007 André-Aisenstadt Chairs



Noga Alon (Tel Aviv)

Paul Seymour (Princeton)

Richard Stanley (MIT)

Combinatorial Optimization

THE second semester of 2006 at the CRM will be devoted to Combinatorial Optimization. The organizing committee comprises David Avis (McGill), David Bremner (New Brunswick), Vašek Chvátal (Concordia), Bill Cunningham (Waterloo), Michel Goemans (MIT), Pierre Hansen (HÉC Montréal), Odile Marcotte (UQÀM), and Adrian Vetta (McGill).

Combinatorial optimization, which will be the theme of the second semester of 2006 at the CRM, deals with optimization problems whose feasible sets are "discrete" subsets of n-dimensional Euclidean space. In practice, the membership of a vector in such a feasible set can be determined in polynomial time, and most combinatorial optimization problems can be modeled as integer linear programming problems. Nowadays combinatorial optimization is one of the most important branches of applied mathematics, and it has been applied to transport scheduling, telecommunications planning, timetabling, VLSI chip design and computational molecular biology.

The semester will feature (June 19 – 30, 2006) the SMS – NATO Advanced Study Institute on Combinatorial Optimization: Methods and Applications, which will be organized by Vašek Chvátal (Concordia) and Najiba Sbihi (École Mohammadia d'Ingénieurs, Rabat). It will be primarily geared towards young faculty members, postdocs, and graduate students. The topics covered will include mixed integer programming, cutting plane approaches, generalized congestion games, stochastic combinatorial optimization, facility location problems, combinatorial optimization in VLSI design, convexity and combinatorial optimization, and applications in supply chain management.

The semester will also feature a series of workshops on the most exciting and timely topics in combinatorial optimization. The first one (June 12 – 14, 2006) will be organized by Joseph Cheriyan (Waterloo) and Michel Goemans (MIT). Its theme will be Approximation Algorithms. Most combinatorial optimization problems are NP-hard, that is, they cannot be solved in polynomial time unless P is equal to NP. As a result, for over three decades, researchers have attempted to derive approximation algorithms that provide in polynomial time suboptimal solutions with a guarantee/proof of their degree of suboptimality. In the last fifteen years, there has been major progress on new techniques (deterministic and randomized roundings, semidefinite programming, embedding of finite metric spaces, etc.) for the design and analysis of these approximation algorithms, and also on negative results or the limit to approximability (through probabilistically checkable proofs and the PCP theorem). For example, a recent result of Khot, Kindler, Mossel and O'Donnell (2005) shows that, modulo the unique games conjecture, any improvement to the semidefinite programming based 0.87856-approximation algorithm of Goemans and Williamson (1995) for the maximum cut problem would imply that P is equal to NP. The workshop will include lectures on the latest developments in the field of approximation algorithms, on both the approximability and the inapproximability sides.

Organized by Shie Mannor (McGill) and Adrian Vetta (McGill), the following workshop (August 14 – 16, 2006) will deal with Network Design: Optimization and Algorithmic Game Theory. Mathematical modelling plays a vital role in the understanding of computer and communication networks. It provides insights into the following questions: allocation of network resources, analysis and effects of competitive and/or cooperative agents, Internet protocols, wireless network protocols, network dynamics, queuing systems performance optimization, and network traffic and topology. These models shed light onto fundamental performance limits and trade-offs, and aid in algorithmic and mechanism design. In particular, the problems arising within that context involve game theoretic analyses that rely on traditional combinatorial and linear programming techniques. As the network infrastructure keeps changing and new applications are emerging, the mathematical models themselves must be adapted constantly. The workshop will explore recent developments in the field and especially the relationship between combinatorial optimization and the models used in distributed network design.

The third workshop (September 18 – 22, 2006) is organized by Vašek Chvátal (Concordia). It will focus on Hybrid Methods and Branching Rules in Combinatorial Optimization. Problems of combinatorial optimization (such as SAT, the problem of recognizing satisfiable boolean formulas in the conjunctive normal form) have been the subject of intensive study by two communities of researchers: those in mathematical programming (often classified under "operations research") and those in constraint satisfaction programming (often classified under "artificial intelligence"). Recent years have seen increasing interaction between these two initially separate communities. One of the aims of the workshop is to foster this confluence.

Traditional methods of combinatorial optimization come in two distinct flavours. Heuristic search algorithms (with variants such as simulated annealing, tabu search, genetic algorithms and neural networks) aim at finding quickly a satisfactory even if not necessarily optimal solution ; where these methods leave off, the exact algorithms (typically some variation on the theme of branch-and-bound) proceed, often through much time-consuming work, to find a provably optimal solution. The more recent hybrid methods borrow ideas from both sources. Development of novel hybrid algorithms will be one of the themes of the workshop.

Branching rules are the other theme of the workshop. These rules are an important component of branch-and-bound-based exact algorithms and their choice may have an overwhelming impact on the efficiency of such algorithms. Branching rules

(*continued on page 16*)

Recent Advances in Combinatorics

by François Bergeron (Université du Québec à Montréal)

THE 2007 theme semester "Recent Advances in Combinatorics" will take place at the CRM from January to June 2007. There will be a total of five main workshops and four major schools to introduce junior mathematicians to the most recent developments in these areas. The schedule is organized so as to have two more intensive periods of activities, one from mid-February to mid-March, and another from early May to mid-June.

The mini-workshop "Algebraic Combinatorics Meets Inverse Systems" will inaugurate the semester from January 19 to January 21, 2007. Organized by F. Bergeron (UQÀM), K. Dalili (Dalhousie), and S. Faridi (Dalhousie), this mini-workshop is a continuation of a sequence of successful such events held in Kingston in 2004, Ottawa in 2005, and at the Fields Institute in 2006. The main achievement of these past workshops has been that they have established an on-going dialog between two research communities who had been using different techniques to study similar mathematical problems. The two groups are algebraic combinatorialists who have been studying important and interesting G-modules for finite groups G generated by reflections; and commutative algebraists who have been studying resolutions and inverse systems in Gorenstein Artinian situations. Exploiting these connections, several new and interesting problems are now being studied, and it seems natural to exploit the expertise of both communities.

A school on Statistical Mechanics and Combinatorics (February 12 – 16, 2007) will precede the workshop on the same subject (February 19 – 23, 2007). Both events are organized by M. Bousquet-Mélou (Bordeaux), P. Leroux (UQÀM), T. Guttmann (Melbourne), and A. Sokal (New York). The goal of this minicourse is to introduce the basic methods of enumerative combinatorics and the concepts of statistical mechanics which are at the heart of the interactions between these two areas. The topics presented will include combinatorial species, functional equations, asymptotics, classical models of statistical mechanics (polyominoes and animals, self-avoiding walks, gas models, Ising and Potts models, polymers and percolation), thermodynamic limits, phase transitions and Tutte polynomials. The ensuing workshop will include expository talks as well as presentations on current research on combinatorial problems raised by statistical mechanics. Topics of interest include: enumerative problems related to the classical models of statistical mechanics, algebraic methods related to functional equations, Mayer's theory and graph weights, Potts model on graphs, planar maps and 2-dimensional quantum gravity, and Feynman diagrams.

The workshop "Recent Progress in Combinatorics on Words" (March 12 – 16, 2007), which is organized by S. Brlek (UQÀM), C. Reutenauer (UQÀM), and B. Sagan (Michigan State), will be preceded by a school (March 5 – 9, 2007) where the basic methods of combinatorics on words will be introduced. The topics will cover finite and infinite words, Sturmian words, patterns, codes, noncommutative series and polynomials. The workshop will include expository talks as well as presentations on current research highlighting the flow of ideas between combinatorics on words and several other fields of mathematics and theoretical computer science. In particular, the emphasis will be on the combinatorial, algebraic and algorithmic aspects underlying the manifold interactions with other fields, in particular Number Theory, Theoretical Computer Science and Algebra.

The second concentration period will begin with a school on Macdonald Polynomials which will take place from April 30 to May 4, 2007, before the workshop on Combinatorial Hopf Algebras and Macdonald Polynomials, which is organized by M. Aguiar (Texas A&M), F. Bergeron (UQÀM), N. Bergeron (York), M. Haiman (Berkeley), and S. van Willigenburg (UBC). The goal of this workshop will be to take stock of ongoing work, and of the many rich problems that still need to be addressed. On one side, the recent past has seen a marked deepened interest in the study of graded Hopf algebras, in part because of their fundamental interactions with algebraic combinatorics, but also because of their importance for Theoretical Physics. In particular, it has recently been made apparent that Hopf algebras play a crucial role in the study of renormalization in quantum electrodynamics. On the other hand, they also appear to play a very significant role in the realm of symmetric and quasisymmetric functions, with surprising repercussions in representation theory, algebraic geometry, mathematical physics, and the combinatorics of Macdonald polynomials. From another perspective, there have been a lot of recent developments regarding combinatorial models for Macdonald polynomials and their link to diagonal coinvariant spaces.

A school (May 21 – 25, 2007) and a workshop (May 28 – June 1, 2007) on Interactions between Algebraic Combinatorics and Algebraic Geometry will be organized by F. Bergeron (UQÀM), A. Geramita (Queen's), A. Knutson (UCSD), R. Vakil (Stanford), and S. Faridi (Dalhousie). The emphasis will be on the study of subjects such as the cohomology of Schubert varieties, Hilbert schemes, Gromov – Witten invariants, and their ties with symmetric functions such as Macdonald polynomials, as well as problems of enumerative geometry in the real, complex and tropical contexts.

Finally, the semester will end with a workshop on Real, Tropical, and Complex Enumerative Geometry (June 11 – 22, 2007), which will be organized by V. Kharlamov (Strasbourg) and R. Pandharipande (Princeton).

Financial support from the NSF will be available for American participants. More information on the program and the support available, see

`http://www.crm.umontreal.ca/Combinatorics2007/`

The 2006 CRM – Fields – PIMS Prize: Nicole Tomczak-Jaegermann

by Nassif Ghoussoub (University of British Columbia)

NICOLE Tomczak-Jaegermann, of the University of Alberta, has been awarded the 2006 CRM – Fields – PIMS prize. According to the citation, "*She has made outstanding contributions to infinite-dimensional Banach space theory, asymptotic geometric analysis, and the interaction between these two streams of modern functional analysis. She is one of the few mathematicians who have contributed important results to both areas. In particular, her work constitutes an essential ingredient in a solution by the* 1998 *Fields Medallist W.T. Gowers of the homogeneous space problem raised by Banach in* 1932."

Tomczak-Jaegermann received her Master's (1968) and Ph.D. (1974) degrees from Warsaw University, where she held a position until moving to the University of Alberta in 1983. There she holds a Canada Research Chair in Geometric Analysis. She is a Fellow of the Royal Society of Canada, lectured at the 1998 ICM, and has won the CMS Krieger – Nelson Prize Lectureship. She has served the Canadian and international research community in many ways, including her current position on the BIRS Scientific Advisory Board and previously as a Site Director of PIMS in Alberta.

What is this area of mathematics which has produced two recent Fields medallists (J. Bourgain and T. Gowers) among many other modern prominent mathematical figures, and yet is still so misunderstood by even the most seasoned of mathematicians? The story starts with the 1932 book of Stefan Banach where he laid the foundation of — infinite-dimensional — Banach space theory. It was to be a unifying framework for many problems arising in differential equations and applied fields, but the intellectual curiosity of the customers of the "Scottish Café" in L'vov took over, and the quest for a "classification theory" for infinite-dimensional Banach spaces started soon after. Most problems turned out to be deep and hard and way beyond the reach of the mathematicians of the 30s and 40s. All these questions have now been answered and many solutions had to wait till the end of the century. But while the questions look like mere mathematical curiosities, the techniques developed to answer them turned out to be rich and far-reaching: from convex analysis to combinatorics, and from infinite-dimensional Ramsey theory, to the refined asymptotics of finite-dimensional convex bodies, via the theories of random matrices and of Gaussian processes.

Nicole Tomczak-Jaegermann

Undoubtedly motivated by the structural rigidity of the classical Banach spaces (Hilbert space, L^p-spaces and spaces of continuous functions), S. Banach posed in his book several intriguing problems about the structure of general infinite-dimensional spaces. Are they isomorphic to their own hyperplanes? to their squares or to their cubes? But the most well-known of the lot were undoubtedly the *Schauder basis problem* and the *homogeneous space problem*. Among Nicole Tomczak-Jaegermann's numerous defining contributions to this field, I shall only describe her contributions to these two problems. I will also discuss briefly her more recent work on the *metric entropy*. I will unfortunately not be able to describe her other equally important contributions to *Banach – Mazur distances* between Banach spaces — in particular between the Schatten classes of operators, to her multiple results with H. König [5] of the *best projection constants problem*, her introduction of the seminal concept of *complex convexity* in infinite-dimensional complex spaces, her influential paper with A. Pajor [9] on an important strengthening of the so-called *Sudakov's minoration theorem* in the theory of Gaussian processes, as well as her most recent results with S. Szarek discovering the phenomenon of *finite-dimensional saturation* and solving a number of open problems from the early 1980s. For all that, I refer the interested reader to her encyclopedic 1989 monograph [10] and of course to her published work.

Before going into more specifics, it is worth emphasizing that the quest to solve these classical problems has led to a whole new field of study now known as *Asymptotic Geometric Analysis*. Initiated and developed by V. Milman and eventually by many others, this new area of research calls for a deeper understanding of infinite-dimensional phenomena via the analysis of various functions of an arbitrarily large number of free variables, as well as certain geometric objects that are determined by an infinitely growing number of parameters. This in turn led to spectacular developments in the so-called *asymptotic theory of convex bodies*, which is roughly concerned with geometric and linear properties of finite-dimensional objects, and the asymptotics of their various quantitative parameters as the dimension tends to infinity.

Results developed in two opposite — yet equally striking — directions. The "optimistic" side was triggered by an early spectacular result of A. Dvoretsky: *Every Banach space of sufficiently large dimension contains a subspace that is almost isometric to Hilbert space* (ℓ_2^k) *of a given dimension* k. In other words, one can find in any n-dimensional convex body a central section of dimension $\log(n)$ which is arbitrarily close to a Euclidean ball. This eventually led to a large number of surprising results, the spirit of which being that certain structures get better and

better as the dimension grows to infinity. The fact that most of these results can be explained by the *concentration of measure phenomenon* started with the exceptional insight of V. Milman, who subsequently developed the concept further in collaboration with M. Gromov and others (e.g., see [7]), leading to equally remarkable results in geometry and combinatorics. This effort was taken up by M. Talagrand and others in the 90s with great results and striking applications to probability and information theory.

The *pessimistic side* was mostly triggered by Gluskin's result who used probabilistic methods to randomly select certain "pathological" projections of the n-dimensional octahedron (the unit ball in ℓ_1^n). These new objects were then superposed by extremely clever techniques for gluing finite-dimensional spaces — initiated by J. Bourgain, S. Szarek, N. Tomczak-Jaegermann and many others — to construct exotic infinite-dimensional counterexamples to several long standing problems, some of which are described below.

I. The Schauder basis problem:
Does every Banach space have a basis?

This problem was of course solved negatively by P. Enflo in the 1970s when he constructed a Banach space without the *approximation property*, and therefore computations in such a space cannot be summarily reduced to manipulating finite-dimensional objects, or finite rank operators. In the 1990's, Nicole Tomczak-Jaegermann and her collaborator P. Mankiewicz went way beyond that particular construction, as they developed an ingenuous method to build such counterexamples in a generic way starting from any non-Hilbertian space. They proved the following

Theorem 1 ([6]). *If X is a Banach space not isomorphic to Hilbert space, then $\ell_2(X)$ has necessarily a quotient space which itself contains a subspace with no Schauder basis.*

Recall that if $(X_n)_n$ is a sequence of Banach spaces, their ℓ_2-sum, $(\bigoplus X_n)_{\ell_2}$, is then the Banach space of all sequences of vectors $z = (z_n)$, with $z_n \in X_n$ for all n, such that $\|z\|_{\bigoplus X_n} = (\sum_n \|z_n\|^2_{X_n})^{1/2} < \infty$. If $X_n = X$ for all n, we then write $\ell_2(X)$ instead of $(\bigoplus X)_{\ell_2}$.

In other words, spaces without a Schauder basis can now be constructed in just three canonical operations starting from an *arbitrary* Banach space X not isomorphic to Hilbert space. Such spaces are of the form $Z = (\bigoplus Z_n)_{\ell_2}$, where Z_n are *finite-dimensional* quotients of subspaces of $\ell_2(X)$. It should be noted that this theorem is amazingly sharp, in the sense that starting with $\ell_2(X)$ — as opposed to X itself — is necessary, since W.J. Johnson had constructed earlier a Banach space X not isomorphic to Hilbert space, all of whose quotients of subspaces do have a basis.

More remarkable are the techniques used for such a construction. They consist of building infinite-dimensional spaces by properly gluing finite-dimensional ones which are themselves obtained by probabilistic methods for selecting appropriate "random quotients." This line of study was initiated by Gluskin who considered random projections of the n-dimensional octahedron (the unit ball in ℓ_1^n) and proved that the diameter of the Banach – Mazur compactum of n-dimensional normed spaces is of order n. The first one to use finite-dimensional random quotients of ℓ_1^n in an infinite-dimensional construction is J. Bourgain who used it to construct a real Banach space that admits two nonisomorphic complex structures.

II. Banach's homogeneous space problem:
Is Hilbert space the only homogeneous Banach space? i.e., is it the only one that can be isomorphic to all of its infinite-dimensional subspaces[1]?

Now we know that the answer to this question of Banach is affirmative, thanks to independent and remarkably complementary contributions by T. Gowers on one hand, and by N. Tomczak-Jaegermann and her student R. Komorowski on the other. The first obvious difficulty in attacking the homogeneous space problem is the lack of information on the uniform boundedness of norms of the isomorphisms. Even up to this day no direct proof is known of the fact that X being homogeneous, must imply that X is *uniformly* isomorphic to all of its infinite-dimensional subspaces, as is the case for Hilbert space which X is supposed to be after all. However, the breakthrough came when N. Tomczak-Jaegermann and R. Komorowski proved that much can be said if the space has an *unconditional basis*: that is a basis $\{z_i\}$ such that for some $C > 0$ we have for any scalar $\{a_i\}$, and any choice of signs, $\{\varepsilon_i\}$, that $\|\sum_i \varepsilon_i a_i z_i\| \leq C\|\sum_i a_i z_i\|$.

Theorem 2 ([4]). *Let X be a Banach space with an unconditional basis, then either X contains a Hilbertian subspace or otherwise it must contain a subspace without an unconditional basis.*

An immediate corollary is the following curious conditional result: If X is a homogeneous Banach space not isomorphic to a Hilbert space, then X cannot have an infinite-dimensional subspace with an unconditional basis. This curiously made a connection with another famous question coming from the 50s: *Does every infinite-dimensional Banach space have an infinite-dimensional subspace with an unconditional basis?*

This question had however received — around the same time — a negative answer by T. Gowers and B. Maurey, via a breakthrough construction that opened a whole new understanding of infinite-dimensional phenomena. This new understanding very fast led to negative solutions for several other problems open for decades, such as the *hyperplane problem* of Banach mentioned above, the *distortion problem* solved by E. Odell and Th. Schlumprecht in [8], as well as many other long-standing open problems. Actually, the Gowers – Maurey space X_0 has a stronger property: no subspace of X_0 is a topological direct sum of two infinite-dimensional Banach spaces.

[1] Here and throughout, all subspaces are assumed to be closed.

Equivalently, given any two infinite-dimensional subspaces Z and W of X_0, we necessarily have

$$\inf\{\|z-w\| : z \in Z, w \in W, \|z\| = \|w\| = 1\} = 0\,.$$

That is, the unit spheres of any two infinite-dimensional subspaces almost intersect. Such a space X_0 is called *hereditarily indecomposable* (an H.I. space). Moreover, they proved the following.

Theorem 3 ([3]). *A hereditarily indecomposable Banach space is not isomorphic to any proper subspace of itself.*

In other words, these spaces are essentially the counterpart of homogeneous spaces. Note also that an H.I. space cannot have an infinite-dimensional subspace with an unconditional basis, since otherwise, such a subspace would be a direct sum of the span of the even elements of the basis and the span of the odd elements. However, the opposite implication is clearly false since there exist spaces which can be decomposed but still have no subspace with an unconditional basis. Many interesting examples of spaces having these and related properties were eventually constructed by Gowers – Maurey, Odell – Schlumprecht, and Argyros and his coauthors, but the precise connection between subspaces with unconditional basis and H.I. subspaces was finally clarified by the spectacular structural dichotomy proved by Gowers in 1993. In particular, it provided the last missing piece in the solution of the homogeneous space problem.

Theorem 4 ([2]). *Every infinite-dimensional Banach space either has an infinite-dimensional subspace with an unconditional basis or has a hereditarily indecomposable subspace.*

The theorem is actually a consequence of a general combinatorial result, which is, in a sense, a vector space analogue of infinite versions of Ramsey theorem.

Once all these results were proved, the solution to the homogeneous space problem is now simple. By the theorem of Tomczak-Jaegermann and R. Komorowski (Theorem 2), a homogeneous space X not isomorphic to Hilbert space cannot have an infinite-dimensional subspace with an unconditional basis. By Gowers dichotomy theorem, it must contain an H.I. subspace, and hence X itself must be H.I. since it is homogeneous. But then, Theorem 3 of Gowers – Maurey says that it cannot then be isomorphic to any proper subspace of itself, which means that X is not homogeneous after all.

III. The finite-dimensional isomorphic version of the homogeneous space problem

It is well known that all finite-dimensional Banach spaces of the same dimension (say n) are isomorphic to Euclidean space ℓ_2^n. However, the isomorphism constants can vary wildly, and so one can ask the following finite-dimensional version of the homogeneous space problem: *For $0 < \alpha < 1$ and $K \geq 1$ does there exist $f(\alpha, K) > 0$ such that an n-dimensional space X is necessarily $f(\alpha, K)$-isomorphic to Euclidean space ℓ_2^n, whenever all of its $[\alpha n]$-dimensional subspaces are K-isomorphic*?

This question is an isomorphic finite-dimensional version of two questions from Banach's book. The first one regards an n-dimensional symmetric convex body all of whose k-dimensional sections are affinely equivalent, which was almost completely solved by Gromov in his doctoral thesis ($K = 1$ in the above question). The second one was the homogeneous space problem discussed above.

A positive answer to the above question was proved for sufficiently small α in 1987 by J. Bourgain. In 1989, N. Tomczak-Jaegermann and P. Mankiewicz managed to prove the result for all α, with a "reasonable" function $f(\alpha, K)$. Actually, $f(\alpha, K) \leq cK^{3/2}$ for $0 < \alpha < \frac{2}{3}$, and cK^2, for $\frac{2}{3} \leq \alpha < 1$, where c is a constant only depending on α. Both solutions rely again on the study of random quotients of normed spaces already mentioned above. We note that even though the method for constructing specific convex bodies from random projections of polytopes was initiated by Gluskin in 1981, the consideration of random quotients in a general form started with the above results of J. Bourgain and P. Mankiewicz – N. Tomczak-Jaegermann, and its study was eventually developed jointly by the last two authors in a series of papers over the years.

IV. The metric entropy problem

If K and B are two subsets of a vector space (or just a group, or even a homogeneous space), the *covering number* of K by B, denoted $N(K, B)$, is the minimal number of translates of B needed to cover K. Similarly, the *packing* number $M(K, B)$ is the maximal number of disjoint translates of B by elements of K. The two concepts are closely related and we have

$$N(K, B-B) \leq M(K, B) \leq N(K, (B-B)/2)\,.$$

If now B is the unit ball of a normed space and K a subset of that space (the setting and the point of view functional analysts usually employ), these notions reduce to considerations involving the smallest ϵ-nets or the largest ϵ-separated subsets of K.

Besides the obvious geometric framework, packing and covering numbers appear naturally in several fields of mathematics, ranging from classical and functional analysis, through probability theory and operator theory, to computer science and information theory (where a code is typically a packing, while covering numbers quantify the complexity of a set). As with other notions related to convexity, an important role is often played by considerations involving duality.

In an operator-theoretic context, one considers the so-called *entropy numbers of an operator* $u\colon X \to Y$ where X and Y are Banach spaces. They are defined as

$$e_n(u) = \inf\{\varepsilon : N(uB_X, \varepsilon B_Y) \leq 2^{n-1}\}\,.$$

These numbers are used to quantify compactness properties of the operator and one can easily see that u is a compact operator if and only if $\lim_n e_n(u) = 0$. Now a classical theorem of Schauder states that u is a *compact operator* if and only if its

adjoint u^* is compact, which readily means that the limiting behaviours of the sequences $e_n(u)$ and $e_n(u^*)$ are similar. In 1972, Pietsch asked several specific questions regarding *entropy numbers* and *duality*. Roughly speaking, do these dual entropy numbers always obey similar asymptotic behaviours? For example, is it true that $\{e_n(u)\}$ belongs to the space ℓ_p (for some $1 \leq p < \infty$) if and only if $\{e_n(u^*)\}$ does? The strongest version of Pietsch's conjectures can also be formulated in the language of covering numbers in the following way:
There exist numerical constants $a, b \geq 1$ such that for any dimension n and for any two centrally symmetric convex bodies K, B in $\mathbb{R}^n$ one has

$$b^{-1} \log_2 N(B^\circ, aK^\circ) \leq \log_2 N(K, B) \leq b \log_2 N(B^\circ, a^{-1}K^\circ)\,?$$

Here $A^\circ := \{u \in \mathbb{R}^n : \sup_{x \in A} \langle x, u \rangle \leq 1\}$ denotes the polar body of A.

This conjecture is still open in its full generality. However, the question about the "global" behaviour of entropy numbers was settled positively in 1987 by N. Tomczak-Jaegermann in the special but central case, when either the domain or target space is a Hilbert space, and more generally by J. Bourgain, A. Pajor, S. Szarek and N. Tomczak-Jaegermann in 1989, in the much more general situation where one of the spaces is of type p, for some $p > 1$. Such spaces also comprise all ℓ_p and L_p-spaces (whether classical or noncommutative) for $1 < p < \infty$, as well as all uniformly convex and all uniformly smooth spaces. In this case, the constants a, b depend only on p and they are uniformly bounded if p stays away from 1 and ∞. More recently, the strongest version of Pietsch's conjecture stated above, was established by Artstein, Milman and Szarek in 2003, again in the case when one of the spaces is a Hilbert space (equivalently, when the convex body is an ellipsoid). N. Tomczak-Jaegermann joined efforts with them in 2004 (see [1]) to establish the conjecture when one of the spaces is of type $p > 1$, and to develop the theory still further.

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Prizes

CAP/CRM Prize

John Harnad

The CRM and the Canadian Association of Physicists (CAP) are pleased to announce that the 2006 CAP/CRM Prize in Theoretical and Mathematical Physics is awarded to John Harnad (Concordia University and CRM), for his outstanding contributions to the theory of integrable systems with connections to gauge theory, inverse scattering and random matrices. Professor Harnad will receive the prize during the 2006 CAP Congress at Brock University in St. Catharines, Ontario, from June 11 – 14, where he will give a plenary lecture.

CRM-SSC Prize

Jeffrey Rosenthal

Jeffrey Rosenthal, Professor in the Department of Statistics at the University of Toronto, is the 2006 winner of the CRM-SSC Prize, which is jointly awarded each year by the Centre de recherches mathématiques and the Statistical Society of Canada to a Canadian statistician in recognition of outstanding contributions to the discipline during the recipient's first 15 years after earning a doctorate. Dr. Rosenthal is one of the leaders in the development of Markov chain Monte Carlo methods. The announcement of the 2006 CRM-SSC Prize was made at the University of Western Ontario in London, site of this year's SSC Annual Meeting.

The 2005 – 2006 André-Aisenstadt Prize

by Iosif Polterovich (Université de Montréal)

THE 2005 – 2006 André-Aisenstadt Prize was awarded jointly to Iosif Polterovich (Université de Montréal) and Tai-Peng Tsai (University of British Columbia) on April 28, 2006. After obtaining a Master's degree from Moscow State University in 1995, Dr. Polterovich obtained his doctorate from the Weizmann Institute in 2000. Following postdoctoral positions at the CRM, MSRI and the Max Planck Institute, he became a faculty member at the Université de Montréal in 2002. Dr. Polterovich works in geometric spectral theory, his broad variety of results being notable for both their importance and novelty. Perhaps most exciting was Polterovich's announcement in 2000 of an "explicit" formula for the heat invariants of a Riemannian manifold; these geometric invariants had been studied for more than fifty years, yet Polterovich presented them in a striking and useful way, which will undoubtedly be central to much forthcoming research by him and others. Tai-Peng Tsai discusses here his main research interests.

GEOMETRY and dynamics on Riemannian manifolds are closely linked to the properties of eigenvalues and eigenfunctions of the Laplace operator. This relationship strongly manifests itself in *spectral asymptotics*. In this article I focus on two subjects: lower bounds for the spectral function and the remainder in Weyl's law [2,3], and heat kernel asymptotics [6,7]. These problems are in a sense opposite, because the first is "direct" (spectral properties are recovered from geometry and dynamics), and the second is "inverse" (geometric information is extracted from the spectrum). At the same time, the two questions are related because heat asymptotics contributes to the growth of the error term in Weyl's law.

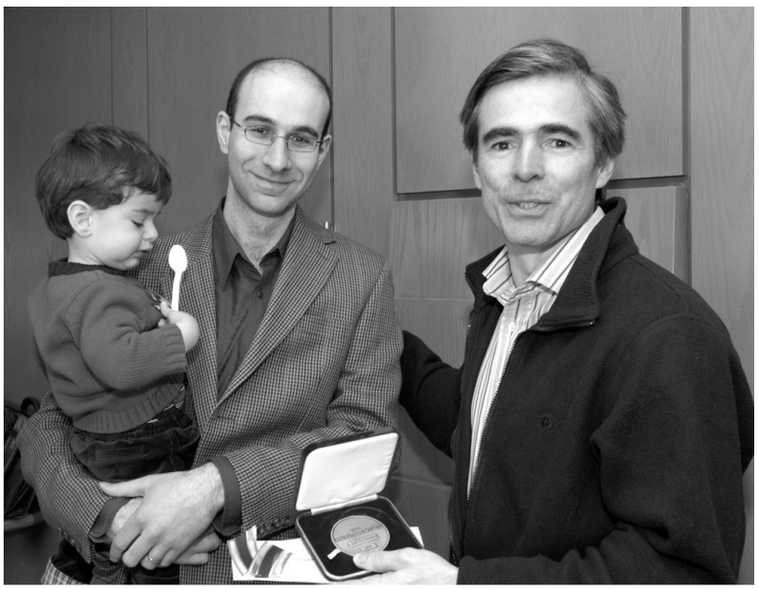
In order to give you a flavour of spectral asymptotics, let me start with an "elementary" example. Surprisingly, it has little to do with either geometry or analysis, but rather belongs to the realm of number theory. Consider the circle of radius r centered at the origin of the Cartesian plane. Let $\mathcal{N}(r)$ be the number of integer points contained inside the circle. What happens to $\mathcal{N}(r)$ as $r \to \infty$? It was proved by Gauss that $\mathcal{N}(r) = \pi r^2 + \mathcal{R}(r)$, where the main term is the area of the unit disk and the remainder satisfies $\mathcal{R}(r) = O(r)$. The celebrated "Gauss's circle problem" is to sharpen the error estimate. Hardy conjectured that $\mathcal{R}(r) = O(r^{1/2+\epsilon})$ for any $\epsilon > 0$. This conjecture has been open for about a century. Hardy and Landau also proved that $\mathcal{R}(r) = \Omega(\sqrt{r})$, where $\Omega(\sqrt{r})$ means $\limsup_{r\to\infty} |\mathcal{R}(r)|/\sqrt{r} > 0$. Hence, the bound proposed by Hardy is the best possible.

How is this example related to spectral asymptotics on manifolds? Let $N(\lambda)$ be the number of eigenvalues less than λ^2. $N(\lambda)$ is called the *counting function*. Consider now a flat square torus of area one. The eigenvalues of the Laplacian on this surface are given by $4\pi^2(k^2 + l^2)$, where k, l run through the integers. Therefore, $N(\lambda)$ coincides up to a constant multiple with the function $\mathcal{N}(r)$ from the Gauss's circle problem! In fact, Gauss's asymptotic formula for the number of integer points is a special case of *Weyl's law*: on any n-dimensional Riemannian manifold M of volume $\mathrm{Vol}(M)$, the eigenvalue counting function satisfies

$$N(\lambda) = \frac{\mathrm{Vol}(M)\lambda^n}{(4\pi)^{n/2}\Gamma(n/2+1)} + R(\lambda)\,,$$

where the remainder $R(\lambda) = O(\lambda^{n-1})$. There are various physical interpretations of Weyl's law, see [4,8].

The counting function can be constructed using a local object — the *spectral function*: $N_{x,y}(\lambda) = \sum_{\sqrt{\lambda_i}\le\lambda} \phi_i(x)\phi_i(y)$. Here x, y are points of the manifold, λ_i are the eigenvalues and ϕ_i are the eigenfunctions that form an orthonormal basis in L^2. On the diagonal $x = y$ we denote $N_{x,y}(\lambda)$ simply by $N_x(\lambda)$. The integral of $N_x(\lambda)$ over the manifold gives precisely the counting function $N(\lambda)$. The function $N_x(\lambda)$ satisfies $N_x(\lambda) = \lambda^n/(4\pi)^{n/2}\Gamma(n/2+1) + R_x(\lambda)$, where $R_x(\lambda) = O(\lambda^{n-1})$. This relation is called a *local Weyl's law*.



Iosif Polterovich, 2005 – 2006 André-Aisenstadt Prize winner, his son, and François Lalonde, CRM Director

A problem of fundamental importance is to estimate $R(\lambda)$, as well as $R_x(\lambda)$ and $N_{x,y}(\lambda)$. The bound $O(\lambda^{n-1})$ proved by Levitan, Avakumovic and Hormander is sharp and attained on a round sphere. Duistermaat and Guillemin proved that if the manifold has "not too many" closed geodesics then $R(\lambda) = o(\lambda^{n-1})$. Upper bounds for the remainder in Weyl's law were studied intensively and various improvements were obtained under additional geometric and dynamical assumptions, see [2,3] and references therein. In particular, a two-term

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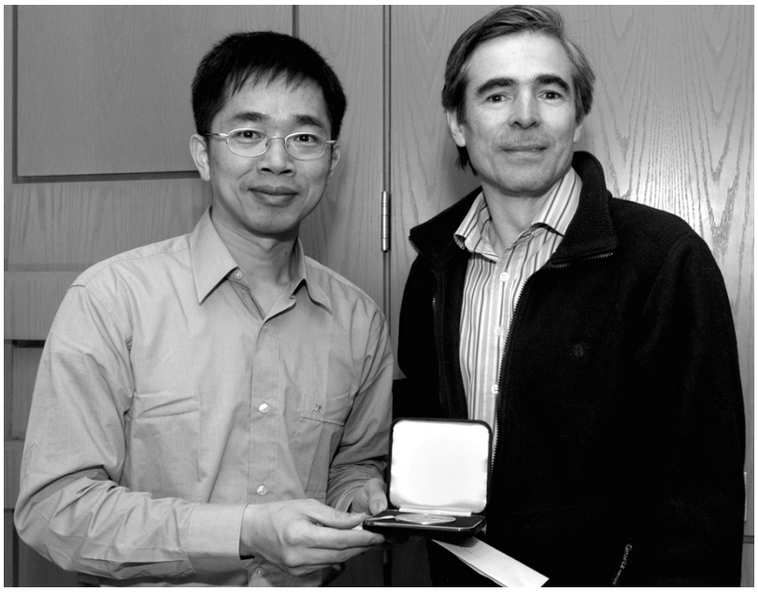
The 2005 – 2006 André-Aisenstadt Prize

by Tai-Peng Tsai (University of British Columbia)

THE 2005 – 2006 André-Aisenstadt Prize was awarded jointly to Tai-Peng Tsai (University of British Columbia) and Iosif Polterovich (Université de Montréal) on April 28, 2006. After completing his B.Sc. at the National Taiwan University in 1991, Dr. Tsai obtained his Ph.D. from the University of Minnesota in 1998 under the supervision of Vladimir Šverák. Following three postdoctoral years at the Courant Institute and a further year at the Institute for Advanced Study, he became an assistant professor at UBC in 2002. Dr. Tsai is an outstanding researcher in nonlinear partial differential equations. In recent work with Kang and Gustafson, he obtained the optimal partial regularity result for the incompressible Navier – Stokes equation. Even more remarkably, he proved the nonexistence of self-similar blow-up solutions (as proposed by Leray in 1934) with finite local energy in three dimensions. With several coauthors, Tsai has also embarked on a deep and detailed study of long-time asymptotics in nonlinear Schrödinger equations. These papers reveal a variety and subtlety of behaviours, and are becoming quite influential. He discusses here his main research interests.

MY research lies in two areas of partial differential equations. One is the regularity problem of the 3-dimensional incompressible Navier-Stokes equations (INS). Another is the stability problem of solitary waves for dispersive equations.



Tai-Peng Tsai, 2005 – 2006 André-Aisenstadt winner, and François Lalonde, CRM Director

The regularity problem of 3D INS asks whether solutions of INS with smooth data may develop a singularity in finite time. Leray conjectured in 1934 [11] the existence of singular solutions with self-similar profile. Nečas, Růžička and Šverák [14] proved that such solutions with finite *global* energy do not exist. I proved subsequently [16] that such solutions with finite *local* energy do not exist either. It is interesting to note that solutions (not necessarily self-similar) with finite local energy allow blow-up at (x_0, t_0) at the self-similar rate

$$|u(x,t)| \sim \frac{C}{|x-x_0| + \sqrt{t_0 - t}}, \quad C > 1, \tag{1}$$

called Type-I blow-up. All other known regularity criteria assume, in average, $|u(x,t)| \ll (|x-x_0| + \sqrt{t_0-t})^{-1}$. Type-I blow-up is quite common for other equations and is the first major enemy one faces in attacking the regularity problem. After another paper with Šverák on spatial decay of stationary solutions of INS [15], I shifted my focus to dispersive equations. Recently I started working on this problem again with Gustafson and Kang, and have improved the known regularity criteria in borderline cases [3, 5] in terms of dimension-free rescaled space-time integrals of the velocity or the vorticity.

My work on dispersive equations began with the work with J. Fröhlich and H.-T. Yau [2] on the semi-classical limit and nonlinear wave operator for Hartree equations. I later published a series of work with Yau [18–21], giving a complete picture of the asymptotic dynamics of small solutions $\psi(t,x) \colon \mathbb{R} \times \mathbb{R}^3 \to \mathbb{C}$ of nonlinear Schrödinger equations (NLS) with a linear potential which admits two eigenvalues:

$$i\partial_t \psi = (-\Delta + V)\psi + \alpha|\psi|^2\psi, \quad \alpha = \pm 1\,. \tag{2}$$

Along the linear eigenfunctions there are two families of nonlinear bound states bifurcating from 0. We proved the stability of nonlinear ground states and the instability of nonlinear excited states. We constructed solutions converging to excited states and also classified all possible asymptotic profiles even when the dispersive component dominates. Later on, I extended the stability and instability results to the many eigenvalue case [17].

The above mentioned work assumes localized initial data, which is also assumed in most of the literature. It ensures the fast decay of the dispersion but is instantaneously lost, unlike the natural energy space H^1 for NLS. In a joint project with Gustafson and Nakanishi [7, 10], we studied the same equation (2) with (nonlocalized) H^1 data. We obtained the first H^1 asymptotic stability results for NLS solitary waves when the linear part admits one or two eigenvalues. In the one eigenvalue case, we describe the dynamics in a neighborhood of 0 for nonlinearities that can be very general and include borderline power nonlinearities and Hartree-type nonlinearities. In the two eigenvalue case, we prove the stability of ground states for cubic nonlinearity.

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Grandes Conférences du CRM

Soucieux de répondre aux attentes d'un public curieux de comprendre les développements marquants des sciences mathématiques, le CRM a lancé au printemps 2006 les « Grandes Conférences du CRM ». Elles mettront en vedette des conférenciers expérimentés, capables de communiquer la beauté et la puissance de la recherche mathématique de pointe dans un langage accessible à tous. Jusqu'ici deux conférenciers on été présentés : Jean-Marie De Koninck et Ivar Ekeland.

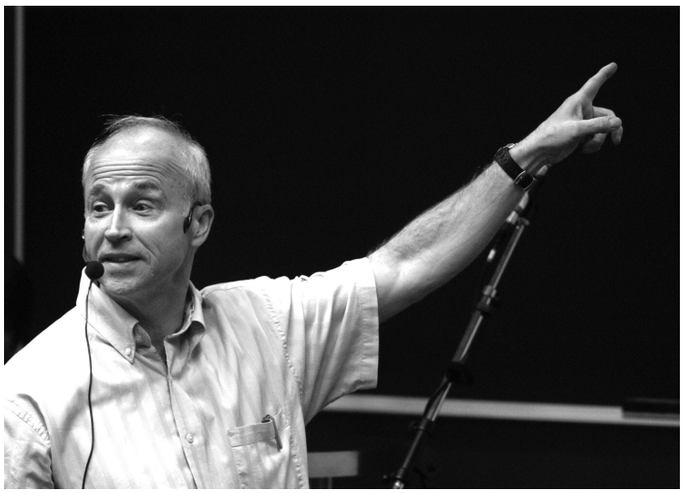
Jean-Marie De Koninck

par Chantal David (Université Concordia)

La première des « Grandes Conférences du CRM »a eu lieu le mercredi 29 mars à 20 heures au Pavillon Jean-Coutu de l'Université de Montréal. Le conférencier était Jean-Marie De Koninck de l'Université Laval, un mathématicien bien connu pour ses travaux mathématiques, mais aussi pour son implication dans plusieurs projets destinés à faire connaître les mathématiques au grand public. Jean-Marie De Koninck vient d'ailleurs de recevoir le prix de Scientifique de l'année 2005 de la Société Radio-Canada qui souligne ainsi depuis près de vingt ans l'engagement et les réalisations d'une personnalité scientifique québécoise. C'est la première fois qu'un mathématicien remporte un tel honneur. Le jury a voulu récompenser le projet « Sciences et mathématiques en action » (SMAC). Le site internet et le spectacle humoristique « Show Math », qui a été présenté à plus de 3 000 élèves du secondaire, démontrent aux jeunes l'utilité des mathématiques dans la vie de tous les jours. Ce prix s'ajoute à de nombreuses distinctions reçues par Jean-Marie au cours de sa carrière, dont l'Ordre du Canada en 1994 et de l'Ordre national du Québec en 1999, ainsi que le prix Adrien-Pouliot 2004 de la Société Mathématique du Canada.

Jean-Marie De Koninck est une personnalité bien connue du grand public québécois : les téléspectateurs de Radio-Canada l'entendent commenter les épreuves de natation aux Jeux Olympiques depuis 1976, et c'est aussi à lui qu'on doit Opération Nez Rouge, lancée en 1984 à Québec, pour prévenir les accidents de la route dus à l'alcool pendant le temps des Fêtes. Ce programme original a fait des petits partout au Québec, et a maintenant franchi les frontières, et s'est implanté notamment en France, en Suisse et au Portugal. Jean-Marie a aussi été animateur et recherchiste pour 29 épisodes de la série télévisée « C'est mathématique ! » (2000 et 2001) diffusée au Canal Z, au Canal Savoir et sur TFO. Lors de ses multiples participations à des émissions de radio ou de télévision, l'enthousiasme de Jean-Marie De Koninck pour les mathématiques réussit toujours à rejoindre le grand public. Grâce à lui, les automobilistes québécois peuvent réfléchir à l'hypothèse de Riemann en manœuvrant dans la circulation matinale…

L'exposé de Jean-Marie a donc eu lieu devant une salle pleine, de plus de 250 personnes, des mathématiciens, bien sûr, mais aussi des étudiants de tous les niveaux (université, cégep, secondaire…) et des scientifiques de tous horizons. Son exposé, intitulé « Quand la réalité déjoue l'intuition », présentait certains problèmes bien connus où l'intuition peut jouer des tours. Par exemple, le fameux « paradoxe des anniversaires ». Si on



Jean-Marie De Konninck

veut réunir assez de personnes pour garantir que deux d'entre elles aient la même date d'anniversaire, il en faut bien sur 367 (n'oublions pas les années bissextiles…) Par contre, si on veut que deux personnes aient le même anniversaire avec une probabilité supérieure à 50 %, il suffit de rassembler 23 personnes. Et pour 60 personnes, la probabilité dépasse les 99 %. Par plusieurs exemples similaires choisis en géométrie, en théorie des probabilités et en théorie des nombres, Jean-Marie De Koninck a ainsi illustré les pièges qui guettent le mathématicien.

Son exposé s'est terminé par un exemple qui occupe toujours les chercheurs en théorie des nombres : une conjecture bien connue de Hardy et Littlewood, datant de 1922, prédit qu'il y a une infinité de premiers jumeaux, c'est-à-dire de la forme p et $p+2$ (comme 5 et 7, ou 11 et 13, ou encore 101 et 103). On peut généraliser cette conjecture aux triplets de premiers, par exemple les premiers de la forme p, $p+2$ et $p+6$, comme 5, 7 et 11, ou encore 41, 43 et 47. On peut aussi généraliser aux quadruplets de premiers, et ainsi de suite. La conjecture des premiers jumeaux généralisée de Hardy et Littlewood englobe tous ces cas. Bien qu'aucun cas ne soit encore démontré, des recherches à l'ordinateur semblent indiquent la validité de cette conjecture.

Un autre conjecture de Hardy et Littlewood affirme que la fonction $\pi(x)$, qui compte le nombre de premiers inférieurs à x, est convexe, c'est-à-dire, qu'il y a toujours plus de premiers dans l'intervalle $[1, y]$ que dans chaque intervalle $[x, x+y]$ pour n'importe quel x. Encore une fois, des recherches à l'ordinateur semblent indiquer clairement la validité de cette conjecture. Mais il a été démontré en 1973 par Hensley et Richards que ces deux conjectures sont incompatibles, et ne peuvent être vraies toutes les deux ! Comment notre intuition peut-elle nous tromper de cette façon ? On peut maintenant démontrer que

si la conjecture des premiers jumeaux généralisée de Hardy et Littlewood est vraie (ce que croient la plupart des experts), il existe alors un intervalle $[x_0, x_0 + 4893]$ qui contient plus de premiers que l'intervalle $[1, 4893]$. Mais on peut aussi estimer empiriquement que x_0, une fois écrit en base décimale, a entre 1 057 et 1 590 chiffres !

Ainsi, Jean-Marie De Koninck a su encore une fois être un ambassadeur des mathématiques auprès des amateurs présents à cette grande conférence, et à la fois intéresser et intriguer ses collègues mathématiciens !

Ivar Ekeland

par Jean LeTourneux (Université de Montréal)

Ivar Ekeland

Le 4 mai 2006, à l'amphithéâtre IBM de l'École des HÉC de Montréal, le professeur Ivar Ekeland prononça, devant un auditoire nombreux et enthousiaste, la deuxième des Grandes Conférences du CRM. Depuis 2003, ce mathématicien éminent dirige le PIMS à Vancouver et occupe une Chaire de recherche du Canada en économie mathématique à l'Université de la Colombie-Britannique. Formé à l'École Normale Supérieure, il enseigna à l'Université de Paris-Dauphine de 1970 à 2003 et en fut le président de 1989 à 1994. Il dirigea également le Centre de Recherche en Mathématiques de la Décision et l'Institut Finance Dauphine. Des prix de l'Académie des Sciences de Paris, de la Société Mathématique de France ainsi que de l'Académie des Sciences de Belgique, sont venus couronner les travaux scientifiques d'Ivar Ekeland.

Soucieux d'atteindre le grand public, il a écrit des ouvrages de vulgarisation et de réflexion, entre autres *Le Calcul, l'imprévu* et *Au hasard*, qui lui valurent le prix Jean-Rostand de l'Association des Écrivains Scientifiques de France et le prix d'Alembert de la Société Mathématique de France. Sa conférence du 4 mai reprenait le titre du dernier paru de ces ouvrages, *Le meilleur des mondes possibles*.

Pour introduire son sujet, Ekeland mit en parallèle deux destins, celui de la Sagouine, l'héroïne d'Antonine Maillet qui se demande pourquoi le monde est dans l'état où il se trouve, et celui d'Archimède qui, à la poursuite de la science pour la science, veut savoir comment est le monde. Le pourquoi et le comment, deux interrogations radicalement différentes qui se sont pourtant croisées à un moment bien précis de l'histoire, quand, en 1744, Pierre Moreau de Maupertuis crut pouvoir répondre aux deux questions à la fois en proposant son principe variationnel de moindre action.

Comme le rappela le conférencier, c'est du côté de la théorie de la lumière qu'il faut chercher l'origine de ce principe. Dès l'Antiquité, Héron d'Alexandrie avait retrouvé la loi de la réflexion en faisant l'hypothèse que la lumière parcourt un trajet de longueur minimale. Au XVIIe siècle, Descartes crut avoir démontré la loi de la réfraction, à tort puisque son raisonnement attribuait à la lumière une vitesse plus grande dans l'eau que dans l'air ! Supposant le contraire, Fermat réussit à tirer cette loi de l'hypothèse que la lumière se déplace de façon à minimiser le temps de parcours.

Pour Maupertuis, la lumière ne suit ni le chemin le plus court, ni le plus rapide : elle emprunte celui qui minimise l'action, une quantité mal définie choisie par Maupertuis de telle sorte qu'il parvint à retrouver la loi de la réfraction en reprenant l'hypothèse erronée de Descartes sur la vitesse de la lumière ! Un succès aussi douteux n'aurait sûrement pas retenu l'attention de la postérité si Maupertuis ne s'était avisé de généraliser son principe de la moindre action pour en tirer toutes les lois de la mécanique classique. Selon ce principe, parmi tous les mouvements possibles la nature choisit celui qui minimise l'action, ce qui semble attribuer un pouvoir de décision à la nature. De là à conclure que ce principe jette un pont entre physique et métaphysique, il n'y a qu'un pas, vite franchi par Maupertuis. N'écrit-il pas :« Lorsqu'on saura que toutes les lois du mouvement sont fondées sur le principe du mieux, on ne pourra douter qu'elles doivent leur établissement à un Être tout puissant et tout sage. » ? Maupertuis connaît une fin misérable. Un ancien ami, Koenig, soutient que Leibniz a énoncé le principe de moindre action avant lui dans une lettre qu'il ne réussit pas à retrouver. La réaction de Maupertuis déclenche une querelle, puis une avalanche de pamphlets. Voltaire s'en mêle et immortalise dans le ridicule Maupertuis et Leibniz avec *L'histoire du docteur Akakia* et *Candide*. Malgré son incompétence en physique et en mathématiques, Maupertuis avait pourtant eu une intuition juste. Il appartiendra à Euler et à Lagrange de définir correctement l'action et le principe variationnel permettant de la rendre stationnaire. La dimension métaphysique se retrouvera au rancart.

Rien dans la physique ne nous permet de penser que le monde a été organisé en vue d'un quelconque destin ou qu'il est meilleur ou pire qu'un autre. À la fin de son exposé, Ivar Ekeland examina rapidement la situation dans d'autres domaines. Il évoqua Darwin, pour qui la perfection n'est pas absolue puisqu'une espèce parfaitement adaptée ici ne l'est pas ailleurs, et l'économiste Hayek pour qui le mieux est l'ennemi du bien. Et il conclut avec beaucoup de sagesse qu'Archimède et la Sagouine auraient intérêt à se rencontrer : Archimède pour encourager la Sagouine à réfléchir par elle-même, la Sagouine pour convaincre Archimède de mieux communiquer et de se pencher sur les vrais problèmes. Un conseil qu'elle ne saurait certes pas adresser à Ivar Ekeland, dont l'exposé riche de réflexions stimulantes constitua un véritable tour de force : pas une seule équation !

Scientific Reports/Rapports scientifiques

A Conference Celebrating the 65th Birthday of Stanislav Molchanov

June 27 – July 1, 2005, CRM
Organizers: Don Dawson (Carleton/McGill), Vojkan Jaksic (McGill), Boris Vainberg (UNCC)

More than fifty persons participated in this conference in honor of Stanislav Molchanov (UNC-Charlotte), a leading probabilist and mathematical physicist who turned 65 in 2005. The topics discussed at the conference were closely related to the research interests (past and present) of Stas Molchanov and covered vast areas of pure and applied mathematics.

The conference opened, on Monday morning, with talks by Barry Simon and his graduate student M. Stoiciu on closely related matters concerning the structure of zeros of orthogonal polynomials. In particular, M. Stoiciu talked about the orthogonal polynomial analog of Molchanov's celebrated result on Poissonian statistics of eigenvalues. Then, two long-term collaborators of Stas, W. Gartner and W. Koenig, gave talks on the parabolic Anderson model. In the afternoon, A. Klein and F. Germinet reported on new and truly spectacular results in the spectral theory of random Schrödinger operators, while Y. Last gave a beautiful review of the structural properties of Anderson type Hamiltonians. The final talk was by A. Soshnikov, whose career began in Moscow under Molchanov's supervision. On the second day, spectral theory, group theory and combinatorics were discussed in the morning (A. Laptev, P. Kuchment, R. Grigorchuk, N. Minami), while the afternoon was devoted to financial (R. Carmona) and applied mathematics (A. Kiselev, M. Fredilin), as well as probablistic interacting particle systems (A. Ramirez).

The talks of Wednesday morning were focused on statistical mechanics (A. Figotin, B. Vainberg, L. Koralov), and the last talk (G. Derfel) dealt with the asymptotics of the Poincaré functions.

On Thursday morning, I. Goldsheid reported on spectacular new results concerning Liapunov exponents, S. Warzel presented a new proof of the celebrated result of Klein on extended states for Anderson model on the Bethe lattice, and K. Khanin dealt with random walks in a quasi-stationary random potential. The first two talks in the afternoon were "random": Y. Godin discussed the random string and L. Bogachev, a former student of Molchanov, the random exponentials. In the afternoon, P. Mueller reported on new results concerning spectral asymptotics of Laplacians on bond-percolation graphs. In the final talk, A. Gordon, another former student of Molchanov, discussed the Cantor spectrum for almost periodic 1D Schrödinger operators.

On the morning of the last day, F. Klopp talked about the exponential sums related to the Kronig-Penney model in a constant electric field, while D. Hundertmark discussed bounds on the spectral shift functions. Then, W. Kirsh gave a very intriguing talk entitled The Draft Constitution of the EU, the Electoral College and Spin Systems. Finally, J. Quinn, a close collaborator of Stas, discussed random generators and various tests for randomness with roots in quantum mechanics. The afternoon talks were given by graduate students (K. Chen, J. Holt, and P. Poulin)

The proceedings of the conference will be published by the CRM.

Fifth Summer School in Quantum Computing

August 1 – 5, 2005, Université de Montréal
Organizers: Alain Tapp (Montréal), André Méthot (Montréal)

It was the second time that the Summer School in Quantum Computing, a now well-established tradition, took place in Montreal. It has been previously held in Toronto, Waterloo and Calgary. The school was targeted to graduate students from the fields of computer science, physics and mathematics. No prior knowledge of quantum computing was assumed. It attracted around sixty students. Many of them were starting graduate studies in one of the Canadian research groups in that field. Many others, working in related fields, attended the school in order to get a basic knowledge of quantum computing.

Quantum computing is a recent research field at the frontier of computer science and physics. It is concerned with using the laws of quantum mechanics to process information. If a quantum computer is ever built, it will have significant impact on computer science in general, but most importantly in cryptology.

The talks covered the following topics: introduction to the computation model of quantum information, quantum cryptography, Grover's search type algorithms, Shor's factoring algorithm, quantum information theory, proofs in the quantum world, error correction and fault tolerant computation, implementation of the quantum computer, non locality, pseudo-telepathy and communication complexity.

The School was happy to welcome distinguished speakers all of whom contributed significantly to the domain. They were: Gilles Brassard (Montréal), Richard Cleve (Waterloo), Claude Crépeau (McGill), Daniel Gottesman (Perimeter Institute), Patrick Hayden (McGill), Peter Høyer (Calgary), Michele Mosca (Waterloo), Barry Sanders (Calgary), Alain Tapp (Montréal), John Watrous (Calgary), and Ronald de Wolf (Amsterdam).

Support came from ICRA, MITACS and CRM.

Équations aux dérivées partielles de grande dimension en sciences et génie

du 7 au 12 août 2005, CRM
Organisateurs : André Bandrauk (Sherbrooke), Michel Delfour (Montréal), Claude Le Bris (CERMICS)

Le programme de cet atelier, qui a attiré une soixantaine de participants, a présenté sur 6 jours 23 conférenciers de renommée internationale autour du grand thème des équations aux dérivées partielles de grande dimension avec la participation multidisciplinaire de chimistes, d'ingénieurs, de mathématiciens, de physiciens et de spécialistes de la finance. Parmi les sujets abordés, mentionnons : les équations cinétiques pour la physique des plasmas, l'équation de Schrödinger à plusieurs corps et celles de Dirac et de Maxwell pour les calculs de structures électroniques et de dynamique moléculaire, les équations de *pricing* pour le calcul d'options en finance mathématique, les équations de Fokker-Planck et de mécanique des fluides pour la simulation de fluides complexes.

Les actes de l'atelier seront publiés dans la collection *CRM Proceedings and Lecture Notes* de l'AMS.

MITACS – MSRI – AFMNet – CRM Workshop on Therapeutic Efficacy in Population Veterinary Medicine

October 19 – 22, 2005, BIRS
Organizer: Fahima Nekka (Montréal)

This workshop was organized by the MITACS BIO5 team around the general theme of therapeutic efficacy in population veterinary medicine at the Banff International Research Station. It brought researchers working in applied mathematics, veterinary sciences, behavioural sciences as well as in microbiology and nutrition. In addition to academic researchers, speakers and participants from other public sectors attended the workshop: Agriculture and Agri-Food Canada as well as the Public Health Agency of Canada. Representatives of Pfizer Animal Health and Elanco Animal Health were present. The lectures covered different aspects relating to animal collective therapy, in particular in swine and poultry, in terms of determinants and outcomes, spanning the areas of: animal behaviour, quantification of feeding behaviour and its relationship with pharmacokinetics, pharmacodynamics and antibiotic resistance, risk assessment in terms of antibiotic use and genetic determinants for antibiotic resistance and its different transfer modes, resistance to infectious diseases, zoonotical borne viruses, identification of contamination sources, characterization of microbial hazards and manure, impact on the environment. A complete portrait of animal behaviour in the context of therapeutic efficacy was drawn. A whole overview of the Canadian Integrated Program for Antimicrobial Resistance Surveillance (CIPARS/PICRA) was given to explain the national program of antimicrobial use in food animals and surveillance system for antimicrobial resistance arising from food animal production. An update of PK/PD analysis in antibiotics was very useful to highlight the role of the prudent use of antibiotics in preserving their effectiveness. A general idea of mathematical approaches used to handle biological complexity was given, with emphasis on the need for collaborative efforts between mathematical sciences and experimental research. The keynote speakers have given their own ideas of possible collaborations with the MITACS' team, in terms of their research interests/expertise and in complement to the current MITACS project. Very interesting discussions took place, always balanced between the different areas of research. Presence of industrial researchers from Pfizer Animal health in particular, allowed gaining a clear idea of the pharmaceutical industry expectations and practices. According to one of the participants, "it was truly encouraging to see genuine collaboration between mathematicians, pharmacologists and animal behaviourists."

Homotopy Theory Conference in Honor of Joe Neisendorfer's 60th Birthday

November 18 – 20, 2005, CRM
Organizer: Octav Cornea (Montréal)

A Conference commemorating Joe Neisendorfer's 60th birthday gathered forty-five participants for two days at the CRM in a pleasant and friendly atmosphere. A number of young mathematicians and graduate students were also present.

Chuck McGibbon (Wayne State) presented a talk entitled "Joe Neisendorfer and his work, an appreciation." The other speakers were Martin Bendersky (Hunter College), Frederick Cohen (Rochester), Brayton Gray (Illinois, Chicago), Steven Halperin (Maryland), Richard Kane (Western Ontario), Ran Levi (Aberdeen), Haynes Miller (MIT), Douglas C. Ravenel (Rochester), Daniel Tanré (Lille), and Laurence Taylor (Notre Dame). These talks covered the development of some central themes in homotopy theory as well as some ramifications towards dynamical systems and singularity theory in Bendersky's talk, towards group theory in Levi's talk, as well as in geometric topology in the talks by Larry Taylor.

The Third Montreal Scientific Computing Days

February 25 – 26, 2006, CRM
Organizers: Anne Bourlioux (Montréal), Paul Tupper (McGill), Thomas Wihler (McGill)

The objective of this conference was to encourage scientific exchange within the scientific computing community in Québec and further afield. It was attended by 70 participants from a variety of universities in Quebec, Ontario, and the USA. The program included two short courses offered by interna-

tional experts, as well as eight contributed lectures of 15 minutes length, mostly by students and postdocs. The course given by Des Higham (Strathclyde) was at a completely introductory level and offered people with no knowledge of the subject an introduction to stochastic differential equations and numerical methods for them. The course of Howard Elman (Maryland) on "Discretization and Solution Algorithms for Problems in Incompressible Fluid Dynamics" was more advanced, being aimed at someone familiar with some more sophisticated techniques in scientific computing such as multigrid and preconditioning.

Activités des laboratoires

Laboratoire de physique mathématique

Random Matrices, Random Processes, and Integrable Systems
June 20 – July 8, 2005, CRM
Organizers: John Harnad (Concordia), Jacques Hurtubise (McGill)

THIS short program tried to emphasize the remarkable connections between two domains that a priori seem unrelated: random matrices (together with associated random processes) and integrable systems. The schedule consisted of two parts. There were eight extended lecture series on related topics, each of one week's duration, having a survey and pedagogical character, aimed primarily at younger researchers entering the field. The afternoon sessions were of "workshop" character, with one hour talks presented on current work in the field.

The lecture series speakers were Mark Adler (Brandeis), Pavel Bleher (Indiana – Purdue), Bertrand Eynard (Saclay), Alexander Its (Indiana – Purdue), Ken McLaughlin (Arizona), Craig Tracy (Davis), Pierre van Moerbeke (Louvain/Brandeis), and Harold Widom (Santa Cruz).

The main topics covered included: spectral theory of random matrices, determinantal ensembles, integral operators in random matrix theory, Dyson processes, Airy, Bessel, sine and Laguerre processes, matrix Riemann – Hilbert methods, applications to large N asymptotics, differential equations for gap distributions and transition probabilities, relations to integrable systems and isomonodromic deformations, growth processes, applications to fluid dynamics and crystal growth, applications to random tilings, random words, random partitions, applications to L-functions, and applications to multivariate statistics.

The schedule and facilities were organized to accommodate a total of approximately 75 participants, coming from eighteen countries around the world, over the three week period of the program, although the steady-state number in any given week was closer to 50. The workshop part of the program also included a number of contributed talks on topics relating closely to the theme of the program. Roughly half the participants were either young researchers, postdoctoral fellows or advanced graduate students, and most of these received partial financing to help cover their travel and/or accomodation expenses.

The Lecture Series part of this workshop will be published in the Springer *CRM Series in Mathematical Physics*, while the workshop proceedings will come out as a refereed Special Issue of *Journal of Physics A: Mathematical and General*.

LaCIM

Colloque Words'05
du 13 au 17 septembre 2005, LaCIM (UQÀM)
Organisateur : Srecko Brlek (UQÀM)

CE colloque est le 5e d'une série de colloques portant sur les mêmes sujets. Il a attiré une centaine de participants. On y a mis l'accent sur le point de vue théorique, en particulier sur les aspects combinatoires algébriques et algorithmiques. La motivation pouvait provenir d'autres domaines comme l'informatique théorique. Le programme comportait six conférences invitées et vingt-huit communications sélectionnées par le comité de programmation.

Les 6 conférenciers invités ont été Arturo Carpi (Pérouse), Maxime Crochemore (Marne-la-Vallée), Michel Mendès-France (Bordeaux), Antonio Restivo (Palerme), Jeffrey Shallit (Waterloo) et Denis Thérien (McGill). On a vivement apprécié l'exposé d'Antonio Restivo qui a explicité des liens entre la combinatoire des mots pure et dure et des applications à la compression de textes, de même que celui de Michel Mendès-France sur l'utilisation du procédé de diagonalisation de Cantor pour la définition de certains mots de dimension deux. L'exposé très clair d'Arturo Carpi sur les répétitions dans les mots a également retenu l'attention.

Les actes du colloque ont été édités par les Publications du LaCIM, dont Srecko Brlek est le responsable, et un numéro spécial de la revue *Theoretical Computer Science A* (*TCSA*) présentant un choix d'articles sera édité par Srecko Brlek et Christophe Reutenauer (UQÀM).

Iosif Polterovich
(*continued from page* 8)

asymptotics for the counting function on manifolds with boundary was proved by Ivrii and Melrose, see [8] for a further discussion.

In a joint work with D. Jakobson [2, 3] we study *lower* bounds for the spectral function and for the error term in local Weyl's law. This direction of spectral estimates is much less explored. The starting point for our research was the bound $R_x(\lambda) = \Omega(\sqrt{\lambda})$ proved by Karnaukh [5] on surfaces of negative curvature. We extend this result and show that on an n-dimensional Riemannian manifold $R_x(\lambda) = \Omega(\lambda^{(n-1)/2})$ at any point x which is not conjugate to itself along any shortest geodesic loop. For example, this is true for every point on a flat 2-torus. Note that in this case $R_x(\lambda) = R(\lambda)$, since the eigenfunctions on a torus are complex exponentials and have absolute value one. Therefore, we get $R(\lambda) = \Omega(\sqrt{\lambda})$ which is the Hardy–Landau bound for the Gauss's circle problem that we discussed above! In other words, the result that we prove may be interpreted as a generalization of the Hardy–Landau bound for the local remainder.

Karnaukh also showed that, if the curvature of a surface satisfies certain pinching condition, the lower bound on $R_x(\lambda)$ can be improved by a logarithmic factor. He used the following idea: Every geodesic loop produces a singularity in the wave equation parametrix and hence contributes an oscillating term to the local remainder. Using the Dirichlet box principle, one can force all these oscillating terms to "point" in the same direction, so that no cancellation occurs. This allows to improve the lower bound.

In [3] we combined this idea with the methods of thermodynamic formalism for hyperbolic flows that permit us to carefully estimate the sums over geodesic loops. This is the main novelty of the paper. As a result, we obtain stronger lower bounds and get rid of the pinching condition. We prove that the spectral function on a compact n-dimensional negatively curved manifold satisfies

$$N_{x,y}(\lambda) = \Omega\big(\lambda^{(n-1)/2}(\log\lambda)^{P(-H/2)/h-\delta}\big)$$

for any $\delta > 0$ and $x \neq y$. Here P denotes the topological pressure, h the topological entropy and H the Sinai–Ruelle–Bowen potential. These quantities are dynamical characteristics of the geodesic flow. One can estimate the power of the logarithm in purely geometric terms. If the sectional curvatures of the manifold lie in the interval $[-K_1^2, -K_2^2]$, then the ratio $P(-H/2)/h \geq K_2/2K_1$, and it equals $\frac{1}{2}$ if the curvature is constant. Our approach illustrates that hyperbolic dynamics can be quite useful in the study of spectral estimates.

A similar lower bound also holds for the remainder $R_x(\lambda)$ on negatively curved manifolds in dimensions two and three. Estimates from below for the error term on higher dimensional manifolds are quite different and can be obtained using the *heat equation asymptotics*. This leads us to the second problem mentioned at the very beginning.

Heat asymptotics became a popular subject in the sixties due to the famous question "Can one hear the shape of a drum?" [4]. It turns out that many geometric invariants that are "audible" (i.e., determined by the spectrum — eigenvalues may be interpreted as frequencies of a vibrating membrane) can be found using the short time asymptotic expansion of the heat trace $\sum_i e^{-t\lambda_i}$. In fact, one such invariant — the volume — can be already found from Weyl's law, since it appears as the coefficient in the main term. The advantage of the heat trace is that, being a smooth function of t, it admits a complete asymptotic expansion and every term is a spectral invariant! The local version of this asymptotics is called the Minakshisundaram–Pleijel expansion: $\sum_i e^{-\lambda_i t}\phi_i^2(x) \sim_{t\to 0^+} (4\pi)^{-n/2}\sum_{j=0}^{\infty} a_j(x)t^{j-n/2}$. The coefficients $a_j(x)$ are *local heat invariants*. For example, $a_1(x)$ gives the scalar curvature, and $\int_M a_1(x)$ in dimension two yields the Euler characteristic (the number of handles of a drum). Using a_2 and a_3 one can show, for instance, that any manifold isospectral to a round sphere of dimension ≤ 6 is isometric to a round sphere. Higher heat invariants contain more information about the metric which, however, is very difficult to extract due to a rapidly increasing computational complexity of $a_j(x)$. There exist various methods for calculation of heat invariants, see [1] and references therein. However, explicit formulas were known only for the first few heat kernel coefficients. Using the Agmon–Kannai commutator expansion, I found a closed formula for all heat invariants on a Riemannian manifold [6]. If $r_x(y)$ is the distance from a point x to a point y, and Δ is the Laplacian, then

$$a_j(x) = (-1)^j \sum_{k=0}^{\omega} \frac{\Gamma(\omega + n/2 + 1)\Delta^{j+k} r_x(y)^{2k}|_{y=x}}{\Gamma(k + n/2 + 1)(\omega - k)!4^k k!(j+k)!}\,.$$

Here ω could be any integer $\geq j$, and this flexibility indicates a remarkable combinatorial structure that is responsible for hidden cancellations in the formula. In [7] this structure is exhibited for the heat invariants on spheres using an approach suggested by Zeilberger. A significant progress in understanding the combinatorics of the heat kernel coefficients was recently achieved by Weingart [10]. Let us go back to the estimates for the remainder in local Weyl's law. Suppose, for simplicity, that $a_1(x) \neq 0$. Then we can show that $R_x(\lambda) = \Omega(\lambda^{n-2})$. If $n \geq 4$ this bound can not be improved by our methods. In order to get some more refined information about the remainder we introduce the *oscillatory error term* $R_x^{\mathrm{osc}}(\lambda)$:

$$N_x(\lambda) = \frac{1}{(4\pi)^{n/2}} \sum_{j=0}^{[(n-1)/2]} \frac{a_j(x)}{\Gamma(n/2 - j + 1)}\lambda^{n-2j} + R_x^{\mathrm{osc}}(\lambda)\,.$$

It is *not* an asymptotic expansion. However, a similar form of Weyl's law is commonly used by physicists: it allows to separate the "smooth" part of the counting function coming from the singularity at zero in the heat asymptotics, and the "fluctuating" part coming from the singularities of the wave equation produced by the geodesic loops. As we prove in [3], the oscillatory remainder satisfies the same lower bound as $N_{x,y}(\lambda)$.

In conclusion, let me mention some questions concerning Weyl's law on negatively curved surfaces, which is the case of particular interest in *quantum chaos* [9]. On a generic negatively curved surface it is believed that $R(\lambda) = O(\lambda^{\epsilon})$ for any $\epsilon > 0$ (we say "generic" to exclude certain arithmetic surfaces with a much faster growth of the remainder). Compared to the lower bound $R_x(\lambda) = \Omega(\sqrt{\lambda})$, this estimate looks quite intriguing, and indicates that huge cancellations should occur when the local remainder is integrated over the surface. It would be interesting to understand the nature of these cancellations. In a work in progress with D. Jakobson and J. Toth we aim to show that $R(\lambda) = \Omega\big((\log\lambda)^{P(-H/2)/h-\delta}\big)$. Such a bound would generalize an old result of Randol for surfaces of *constant* negative curvature, proved by number-theoretical methods. The approach of [3], based on thermodynamic formalism, allows to replace number theory by dynamics, and makes it possible to treat the variable curvature case.

1. P. Gilkey, *Asymptotic formulae in spectral geometry*, Chapman & Hall/CRC, 2003.
2. D. Jakobson and I. Polterovich, *Lower bounds for the spectral function and the remainder in local Weyl's law on manifolds*, Electron. Res. Announc. Amer. Math. Soc. **11** (2005), 71 – 77.
3. _____, *Estimates from below for the spectral function and for the remainder in local Weyl's law*, arXiv:math.SP/0505400.
4. M. Kac, *Can one hear the shape of a drum*?, Amer. Math. Monthly **73** (1966), no. 4, 1 – 23.
5. A. Karnaukh, *Spectral count on compact negatively curved surfaces*, Ph.D. thesis, Princeton Univ., 1996, 1 – 48 (under the supervision of P. Sarnak).
6. I. Polterovich, *Heat invariants of Riemannian manifolds*, Israel J. Math. **119** (2000), 239-252.
7. _____, *Combinatorics of the heat trace on spheres*, Canad. J. Math. **54** (2002), no. 5, 1086 – 1099.
8. Yu. Safarov and D. Vassiliev, *The asymptotic distribution of eigenvalues of partial differential operators*, Transl. Math. Monogr., vol. 155, Amer. Math. Soc., Providence, RI, 1997.
9. P. Sarnak, *Arithmetic quantum chaos*, The Schur Lectures (1992), Israel Math. Conf. Proc., vol. 8, Bar-Ilan Univ., Ramat Gan, 1995, pp. 183 – 236.
10. G. Weingart, *Combinatorics of heat kernel coefficients*, preprint available at `http://www.math.uni-bonn.de/people/gw/`

Combinatorial Optimization
(*continued from page 2*)

are not well understood at present and they may never be well understood. One intriguing template for their design is strong branching, first introduced in Concorde, a computer code for the symmetric travelling salesman problem; another one echoes the theme of hybrid algorithms by letting local search heuristics (such as GSAT or Walksat) pick out promising variables for branching.

On October 10 – 13, 2006, a workshop on Data Mining and Mathematical Programming will take place, organized by Pierre Hansen (HÉC Montréal) and Panos Pardalos (Florida). Data mining is a fast-growing discipline that uses techniques from several subfields of applied mathematics, including operations research and statistics. Most data mining techniques fall into one of the following categories: predictive modelling, clustering, dependency modelling, data summarization, and change and deviation detection (see *Mathematical Programming for Data Mining*: *Formulations and Challenges*, by Bradley, Fayyad and Mangasarian). Many of the problems that arise in these various techniques can be formulated as mathematical programming problems. This workshop will feature applications of exact or heuristic algorithms for solving mathematical programs (linear or nonlinear, convex or nonconvex) to the fundamental problems in data mining, in particular clustering, discrimination and search for relations.

The last workshop of the semester (October 17 – 20, 2006) will be devoted to Polyhedral Computation. It will be organized by David Avis (McGill), David Bremner (New Brunswick), and Antoine Deza (McMaster). The last fifteen years have seen significant progress in the development of general purpose algorithms and software for polyhedral computation (e.g., finding lattice points, enumerating vertices, extreme rays and facets and triangulating polyhedra). Many polytopes of practical interest have enormous output complexity and are often highly degenerate, posing severe difficulties for known general purpose algorithms. They are, however, highly structured and attention has turned to exploiting this structure, particularly symmetry. Initial applications of this approach have permitted computations previously far out of reach, but much remains to be understood and validated experimentally. This workshop will bring together researchers with both theoretical and computational expertise with polyhedral computations.

For more information on the semester and on the support available for visitors, graduate students and postdoctoral fellows, see

`http://www.crm.umontreal.ca/Optimization2006/`

Tai-Peng Tsai
(*continued from page* 9)

Also in the setting of the natural energy space H^1, I obtained with Martel and Merle the orbital stability and asymptotic stability for the sum of N (weakly-interacting) solitons of generalized KdV equations [12]. Very recently, we proved the orbital stability for the sum of N solitary waves of NLS in $\mathbb{R}^n$, $n = 1, 2, 3$, [13]. The only assumption we make for NLS is that the nonlinearity is supercritical when the solution is near zero. We do not make any spectral assumption and we do not assume localized initial data.

In a slightly different direction, Gustafson, Nakanishi and I considered the asymptotic stability for the vacuum state $\psi \equiv 1$ of the Gross – Pitaevskii equation: $\psi\colon \mathbb{R} \times \mathbb{R}^n \to \mathbb{C}$,

$$i\partial_t \psi = -\Delta\psi + (|\psi|^2 - 1)\psi\,, \quad |\psi(t,x)| \to 1 \text{ as } |x| \to \infty\,, \quad (3)$$

which is a dynamical model for superfluids and Bose – Einstein condensates, with nonzero boundary condition at spatial infinity. Previous studies focused on small parameter limits of the solutions and the existence of vortex and traveling wave solutions. We proved the asymptotic stability for the vacuum state for dimensions $n \geq 4$ [8], and also constructed scattering solutions for $n = 2, 3$ [9]. The analysis in low dimensions is very delicate due to a quadratic nonlinear term $|u|^2$ in the equation for $u = \psi - 1$.

In a more geometric direction, I worked with Gustafson and Kang [4, 6] on the regularity problem of Schrödinger flow (also known as Schrödinger map), for which one does not know if finite energy solutions can develop a singularity in finite time. We consider the energy-critical case: $u(t,x)\colon \mathbb{R} \times \mathbb{R}^2 \to \mathbb{S}^2 \subset \mathbb{R}^3$, satisfying

$$\partial_t u = u \times \Delta u\,. \quad (4)$$

For the class of equivariant Schrödinger flows with degree $m \geq 1$ and energy close to a harmonic map energy, we relate the (hypothetical) blow-up to the vanishing of the length scale of the nearest harmonic map. We also show that, when $m \geq 4$, the solution converges locally to a harmonic map at time infinity and does not blow up. The cases $m = 1, 2, 3$ are open.

Another project I worked on is the analytic and numerical study of the spectra of linearized operators for NLS [1].

1. S.-M. Chang, S. Gustafson, K. Nakanishi, and T.-P. Tsai, *Spectra of linearized operators for NLS solitary waves* (submitted, December 2005).
2. J. Fröhlich, T.-P. Tsai, and H.-T. Yau, *On the point-particle (Newtonian) limit of the non-linear Hartree equation*, Comm. Math. Phys. **225** (2002), no. 2, 223 – 274.
3. S. Gustafson, K. Kang, and T.-P. Tsai, *Regularity criteria for suitable weak solutions to the Navier – Stokes equations near boundary*, J. Differential Equations (to appear).
4. _____, *Schrödinger flow near harmonic maps*, Comm. Pure Appl. Math. (to appear).
5. _____, *Interior regularity criteria for suitable weak solutions of the Navier – Stokes equations* (in preparation).
6. _____, *Asymptotic stability of harmonic maps under the Schrödinger flow* (in preparation).
7. S. Gustafson, K. Nakanishi, and T.-P. Tsai, *Asymptotic stability and completeness in the energy space for nonlinear Schrödinger equations with small solitary waves*, Int. Math. Res. Not. **2004**, no. 66, 3559 – 3584.
8. _____, *Scattering for the Gross – Pitaevskii equation*, Math. Research Letters **13** (2006), no. 2, 273 – 285.
9. _____, *Global dispersive solutions for the Gross – Pitaevskii equation* (in preparation).
10. _____, *Asymptotic stability of small NLS solitons in the energy space: the two eigenvalue case* (in preparation).
11. J. Leray, *Sur le mouvement d'un liquide visqueux emplissant l'espace*, Acta Math. **64** (1934), 193 – 248.
12. Y. Martel, F. Merle, and T.-P. Tsai, *Stability and asymptotic stability in the energy space of the sum of N solitons for subcritical gKdV equations*, Comm. Math. Phys. **231** (2002) no. 2, 347 – 373.
13. _____, *Stability in H^1 of the sum of K solitary waves for some nonlinear Schrödinger equations*, Duke Math J. (to appear).
14. J. Nečas, M. Růžička, and V. Šverák, *On Leray's self-similar solutions of the Navier – Stokes equations*, Acta Math. **176** (1996), no. 2, 283 – 294.
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17. _____, *Asymptotic dynamics of nonlinear Schrödinger equations with many bound states*, J. Differential Equations **192** (2003), no. 1, 225 – 282.
18. T.-P. Tsai and H.-T. Yau, *Asymptotic dynamics of nonlinear Schrödinger equations, resonance dominated and dispersion dominated solutions*, Comm. Pure Appl. Math. **55** (2002), no. 2, 153 – 216.
19. _____, *Relaxation of excited states in nonlinear Schrödinger equations*, Int. Math. Res. Not. **2002**, no. 31, 1629 – 1673.
20. _____, *Stable directions for excited states of nonlinear Schrödinger equations*, Comm. Partial Differential Equations **27** (2002), no. 11-12, 2363 – 2402.
21. _____, *Classification of asymptotic profiles for nonlinear Schrödinger equations with small initial data*, Adv. Theor. Math. Phys., **6** (2002), no.1, 107 – 139.

Nouvelles/News

André Bandrauk

UN membre du CRM, le professeur André Bandrauk s'est vu décerner le prix Urgel-Archambault 2005 de l'Acfas. Créé en 1953 pour honorer la mémoire d'Urgel Archambault, directeur-fondateur de l'École Polytechnique de Montréal, ce prix récompense une personne travaillant en sciences physiques, mathématiques ou en génie.

Professeur titulaire au Département de chimie de l'Université de Sherbrooke, André Bandrauk détient une Chaire de recherche du Canada en chimie computationnelle et photonique. Reconnu mondialement pour ses expériences numériques sur les molécules, il a joué un rôle de pionnier dans l'utilisation des lasers comme outils d'étude et de contrôle des phénomènes chimiques.

OUT of the four Canadian invited speakers at the International Congress of Mathematicians (Madrid, 22 – 30 August 2006), three have a close connection with the CRM. Indeed, François Lalonde (Montréal) is the director of this center, Henri Darmon (McGill) is one of its regular members, and Vinayak Vatsal (UBC) received the 2003 – 2004 André-Aisenstadt Prize. The other invited speaker is Michael Shub, from the University of Toronto.

Manjul Bhargava

MANJUL Bhargava (Princeton), who held the André-Aisenstadt Chair at the CRM during the spring of 2006, has been awarded a Clay Research Award in recognition of his discovery of new composition laws for quadratic forms, and of his work on the average size of ideal class groups. The award was presented at the annual meeting of the Clay Mathematics Institute in October 2005 at Oxford University.

LE Canada sera bien représenté lors du 6e congrès international de mathématiques industrielles et appliquées qui se tiendra à Zurich du 16 au 20 juillet 2007. En effet, Ivar Ekeland, directeur du PIMS, a été invité à prononcer la conférence publique du congrès, alors que Barbara Lee Keyfitz, directrice du Fields Institute, et Michel Fortin de l'Université Laval figurent parmi les conférenciers invités.

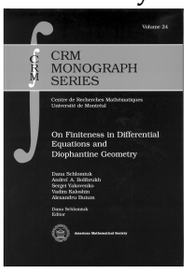
Adam Logan

ADAM Logan won the 2005 World Scrabble Championship in London, England, while he was a visitor at the CRM last fall. Earlier, he had spent the 2002 – 2003 year at CICMA, one of the eight inter-university laboratories of the CRM, which concentrates on number theory and arithmetic geometry. He is now at the University of Waterloo, in the Department of Pure Mathematics.

Born in Ottawa, Logan received an AB degree (1995) and a PhD (1999) from Princeton. He started winning scrabble tournaments by the age of 12. Before his victory in London, he had won the U.S. National Scrabble Championship (1996) and he is the only person to have been the Canadian champion twice, in 1996 and 2005.

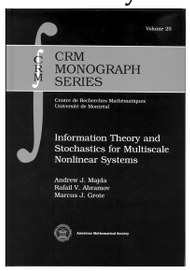
Publications

On Finiteness in Differential Equation and Diophantine Geometry



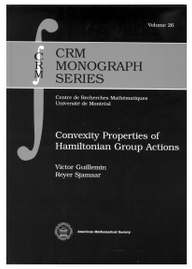
D. Schlomiuk et al.
2005; US$59, AMS members US$47

Information Theory and Stochastics for Multiscale Nonlinear Systems



A.J. Majda et al.
2005; US$39, AMS members US$31

Convexity Properties of Hamiltonian Group Actions



V. Guillemin & R. Sjamaar
2005; US$35, AMS members US$28

Perspectives in Riemannian Geometry

cover nonavailable

V. Apostolov et al. (eds.)
2006; US$85, AMS members US$68

cf. www.crm.umontreal.ca/pub/

Prix de mathématiques André-Aisenstadt
André-Aisenstadt Mathematics Prize

Appel de candidatures

Le Centre de recherches mathématiques (CRM) lance un nouvel appel de candidatures pour le prix de mathématiques André-Aisenstadt visant à reconnaître le talent de jeunes mathématiciens canadiens. Le prix de mathématiques André-Aisenstadt, comprenant une bourse de 3 000 $ ainsi qu'une médaille, souligne des résultats exceptionnels de recherche en mathématiques pures ou appliquées, réalisés par un jeune mathématicien ou mathématicienne canadien.

Le récipiendaire est choisi par le Comité consultatif du CRM. Le prix est normalement accordé annuellement bien qu'il puisse survenir une année où l'on décide de ne pas l'attribuer. Les candidats doivent être citoyens canadiens ou résidents permanents du Canada et avoir terminé leur doctorat depuis sept ans ou moins. Le récipiendaire est invité à prononcer une conférence au CRM et à présenter un résumé de ses travaux pour publication dans le *Bulletin du CRM*.

Les candidatures doivent être parrainées par au moins deux personnes et présentées avant le **1er octobre 2006** au directeur du CRM. Les dossiers devront inclure les documents suivants :
– un curriculum vitæ,
– une liste des publications,
– une lettre justificative,
– au plus quatre tirés à part,
– et un maximum de quatre lettres de référence.

À moins que les candidats ne décident de les retirer, les candidatures non retenues sont automatiquement remises en concours durant les deux années suivantes, à condition que la règle concernant l'obtention du doctorat depuis sept ans ou moins soit toujours satisfaite. Des mises à jour des dossiers peuvent être effectuées.

Créé en 1991, le prix André-Aisenstadt souligne l'excellence de la recherche en mathématiques pures ou appliquées effectuée par de jeunes chercheurs canadiens. Les récipiendaires du prix de mathématiques André-Aisenstadt ont été : Niky Kamran (McGill), Ian Putnam (Victoria), Michael Ward (UBC), Nigel Higson (Penn State), Adrian S. Lewis (Waterloo), Lisa Jeffrey (Toronto), Henri Darmon (McGill), Boris Khesin (Toronto), John Toth (McGill), Changfeng Gui (Connecticut), Eckhard Meinrenken (Toronto), Jinyi Chen (UBC), Alexander Brudnyi (Calgary), Vinayak Vatsal (UBC), Ravi Vakil (Stanford), Iosif Polterovich (Montréal) et Tai-Peng Tsai (UBC).

Prière de soumettre les dossiers au : Directeur, Centre de recherches mathématiques, Université de Montréal, C.P. 6128, succursale Centre-ville, Montréal, QC H3C 3J7, Canada, Fax : (514) 343-2254, `directeur@crm.umontreal.ca`

Call for nominations

The Centre de recherches mathématiques (CRM) solicits nominations for the André Aisenstadt Mathematics Prize awarded to recognize talented young Canadian mathematicians. The André Aisenstadt Mathematics Prize, which recognizes outstanding research achievement by a young Canadian mathematician in pure or applied mathematics, consists of a $3,000 award and a medal.

The recipient is chosen by CRM's scientific advisory committee. The prize is generally awarded yearly, although in a given year the decision may be made not to award it. At the time of consideration, candidates must be Canadian citizens or permanent residents of Canada, and no more than seven years from their Ph.D. The recipient is invited to deliver a lecture at CRM, at a time mutually agreed upon, and to write a brief article on his or her work for publication in the CRM's Bulletin.

Nominations must be submitted by **October 1, 2006**, to the Director of the CRM, by at least two sponsors who are responsible for providing the following information:

- a curriculum vitae,
- a list of publications,
- a covering letter explaining the basis of the nomination,
- up to four reprints, and
- a maximum of four letters of support.

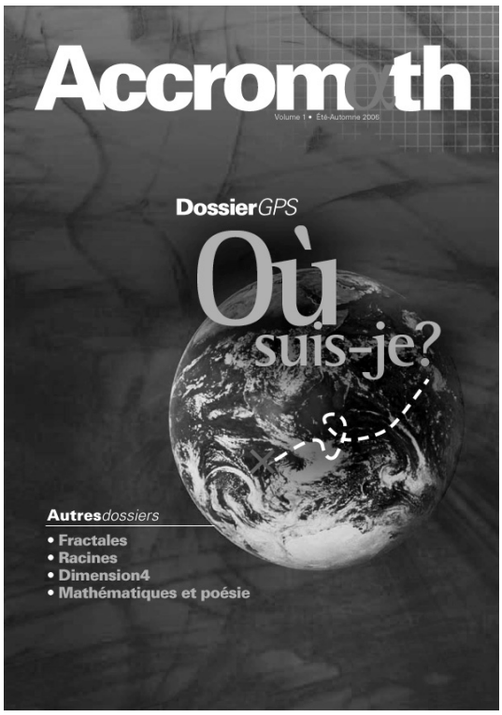
Unsuccessful nominations remain active for two further years if not withdrawn and if the rule regarding the completion of the Ph.D. within seven years is still satisfied. The nominations can be updated if desired.

Created in 1991, the André-Aisenstadt Mathematics Prize is intended to recognize and reward research achievements in pure and applied mathematics by talented young Canadian mathematicians. Previous recipients of the André Aisenstadt Prize: Niky Kamran (McGill), Ian Putnam (Victoria), Michael Ward (UBC), Nigel Higson (Penn State), Adrian S. Lewis (Waterloo), Lisa Jeffrey (Toronto), Henri Darmon (McGill), Boris Khesin (Toronto), John Toth (McGill), Changfeng Gui (Connecticut), Eckhard Meinrenken (Toronto), Jinyi Chen (UBC), Alexander Brudnyi (Calgary), Vinayak Vatsal (UBC), Ravi Vakil (Stanford), Iosif Polterovich (Montréal) and Tai-Peng Tsai (UBC).

Please submit nominations to: Directeur, Centre de recherches mathématiques, Université de Montréal, C.P. 6128, succursale Centre-ville, Montréal, QC H3C 3J7, Canada, Fax: (514) 343-2254, `directeur@crm.umontreal.ca`

Accromαth : les mathématiques comme vous ne les avez jamais vues !

par Alexandra Haedrich (coordonnatrice de l'ISM)



Le 31 mai 2006 lors du « Colloque sur l'enseignement des mathématiques dans la francophonie » à Sherbrooke, l'Institut des sciences mathématiques (ISM) a lancé *Accromαth*, une nouvelle revue mathématique à l'intention des élèves et des enseignants du secondaire et des cégeps. Portant un regard innovateur et rafraîchissant sur les sciences mathématiques, la revue vise surtout à stimuler l'intérêt des jeunes pour les mathématiques et à alimenter leurs enseignants.

S'inspirant de la publication Π *in the Sky* du Pacific Institute for the Mathematical Sciences (PIMS), l'ISM a pris l'initiative de créer une revue en français avec les mêmes objectifs. Grâce à l'appui généreux du directeur du PIMS, Ivar Ekeland, l'ISM et le PIMS ont conclu une entente de collaboration précisant que les deux revues partageraient librement leurs articles, ainsi que leurs idées. Le CRM s'est ensuite joint au projet en tant que partenaire, fournissant une aide logistique et un appui financier importants. C'est enfin le Réseau des centres d'excellence MITACS qui a contribué à la réalisation du projet par l'octroi d'une subvention de démarrage.

Une équipe éditoriale bien chevronnée dans la promotion des mathématiques a conçu, réalisé et peaufiné ce premier numéro. Sous l'impulsion du rédacteur en chef André Ross, du Cégep de Lévis-Lauzon, tous les membres du comité de rédaction ont apporté leur touche à la revue, plusieurs même ayant écrit des articles. (Les membres de ce comité sont France Caron, Louis Charbonneau, Jocelyn Dagenais, Jean-Marie De Koninck, André Deschênes, Christian Genest, Frédéric Gourdeau, Bernard R. Hodgson, Christiane Rousseau). Ensuite, en collaboration étroite avec le graphiste Pierre Lavallée j'ai apporté un soin particulier à l'aspect visuel de la revue.

Accromαth sera diffusée gratuitement dans toutes les écoles secondaires et tous les cégeps du Québec et une version électronique sera disponible sur l'Internet. Deux numéros seront produits par année, dont le prochain en janvier 2007.

Tous ceux et celles qui sont intéressés à contribuer à la revue, en tant qu'auteur, artiste ou critique, sont invités à prendre contact avec moi à ism@math.uqam.ca. Pour en obtenir des copies, voir le site web : `www.accromath.ca`.

Le Bulletin du CRM

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Le Bulletin du CRM est une lettre d'information à contenu scientifique, faisant le point sur les actualités du Centre de recherches mathématiques.

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Le Centre de recherches mathématiques (CRM) de l'Université de Montréal a vu le jour en 1969. Présentement dirigé par le professeur François Lalonde, il a pour objectif de servir de centre national pour la recherche fondamentale en mathématiques et dans leurs applications. Le personnel scientifique du CRM regroupe plus d'une centaine de membres réguliers et de boursiers postdoctoraux. De plus, le CRM accueille d'année en année un grand nombre de chercheurs invités.

Le CRM coordonne des cours gradués et joue un rôle prépondérant en collaboration avec l'ISM dans la formation de jeunes chercheurs. On retrouve partout dans le monde de nombreux chercheurs ayant eu l'occasion de perfectionner leur formation en recherche au CRM. Le Centre est un lieu privilégié de rencontres où tous les membres bénéficient de nombreux échanges et collaborations scientifiques.

Le CRM tient à remercier ses divers partenaires pour leur appui financier de notre mission : le Conseil de recherches en sciences naturelles et en génie du Canada, le Fonds québécois de la recherche sur la nature et les technologies, l'Université de Montréal, l'Université du Québec à Montréal, l'Université McGill, l'Université Concordia, l'Université Laval, l'Université d'Ottawa, ainsi que les fonds de dotation André-Aisenstadt et Serge-Bissonnette.

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