Uncertainty Analysis in Wind Resource Assessment and Wind Energy Production Estimation

Matthew A. Lackner[1], Anthony L. Rogers[2], and James.F. Manwell[3]
Renewable Energy Research Laboratory, University of Massachusetts, Amherst, MA, 01003

This paper presents a mathematical approach to properly account for uncertainty in wind resource assessment and wind energy production estimation. The evaluation of a wind resource and the subsequent estimation of the annual energy production (*AEP*) is a highly uncertain process. Uncertainty arises at all points in the process, from measuring the wind speed to the uncertainty in a power curve. A proper assessment of uncertainty is critical for judging the feasibility and risk of a potential wind energy development. Many current methods for assessing uncertainty either oversimplify the process or make faulty assumptions, leading to erroneous estimates of uncertainty. The approach in this paper yields a more accurate and objective accounting of uncertainty, and therefore better decision making when assessing a potential wind energy site. Three major aspects of site assessment uncertainty are presented here. First, a method is presented for combining uncertainty that arises in assessing the wind resource. Second, uncertainty in wind turbine power output and energy production is characterized. Third, a method for estimating the overall *AEP* uncertainty when using a Weibull distribution is presented. While it is commonly assumed that the uncertainty in the wind resource should be scaled by a factor between two and three to yield the uncertainty in the *AEP*, this work demonstrates that this assumption is an oversimplification, and also presents a closed form solution for the sensitivity factors of the Weibull parameters.

Nomenclature

AEP	=	Annual Energy Production
ARRAY	=	Denotes Array Losses
AV	=	Denotes Availability Losses
c	=	Weibull Scale Factor
CF	=	Capacity Factor
ELF	=	Energy Loss Factor
FOUL	=	Denotes Fouling Losses
k	=	Weibull Shape Factor
h	=	Height
HUB	=	Denotes Hub Height Quantities
LT	=	Denotes Long-Term Quantities or Long-Term Estimation Uncertainty
LT_HUB	=	Denotes Long-Term, Hub Height Quantities
M	=	Denotes Measured Quantities or Measurement Uncertainty
N	=	Number of Measurements
P	=	Power
P_R	=	Rated Wind Turbine Power
$P_W(U)$	=	Wind Turbine Power Curve
$\bar{P}_W$	=	Average Power Output
$p(U)$	=	Wind Speed Probability Density Function
R	=	Range of a Rectangular Distribution

[1] Ph.D. Candidate
[2] Senior Research Fellow
[3] Professor
Department of Mechanical and Industrial Engineering, 160 Governors Drive, Amherst, MA 01003

RERL	=	Renewable Energy Research Laboratory
SA	=	Denotes Site Assessment Uncertainty
SF	=	Sensitivity Factor
U	=	Wind Speed
$\bar{U}$	=	Mean Wind Speed
V	=	Denotes Inter-annual Variability Uncertainty
z_0	=	Log Law Wind Shear Parameter
α	=	Power Law Wind Shear Parameter
Γ	=	Gamma Function
δ	=	Denotes Uncertainty
σ	=	Standard Deviation
*	=	Denotes Absolute Uncertainties (vs. non-dimensional uncertainties)

I. Introduction

Wind energy site assessment gauges the potential for a site to produce energy from wind turbines. When wind energy development is under consideration, a site assessment is undertaken. Specifically, wind energy site assessment is the process of evaluating the wind resource at a potential wind turbine or wind farm location, then estimating the energy production of the proposed project. The wind resource at a site directly affects the amount of energy that a wind turbine can extract, and therefore the success of the venture. The quality of the wind resource is primarily quantified by the mean wind speed at the site, although the turbulence intensity, probability distribution of the wind speed, and prevailing wind direction are also important factors. Once the wind resource has been assessed at a site, the expected annual energy production, *AEP*, of a selected wind turbine is calculated. This calculation combines the expected wind resource with the wind turbine(s) power curve and the expected energy losses in order to estimate how much energy the wind turbine(s) will actually produce at the site. The *AEP* will ultimately help determine the profitability of the undertaking.

The accuracy and precision of the wind resource assessment and *AEP* calculation must also be determined when evaluating a potential site. Wind resource assessment is an uncertain process, and a large number of factors ranging from wind speed measurement errors to the inherent physical variations in the wind contribute to this uncertainty. Overall, these various individual sources of error must all be accounted for to provide an estimate of the total uncertainty of the wind resource. Furthermore, power curves and energy loss terms are uncertain as well. When the wind resource, the power curve, and the energy losses are combined to estimate the *AEP*, the uncertainties from all the factors contribute to an overall *AEP* uncertainty. This uncertainty is critical in estimating the risk associated with the potential venture.

The goal of this paper is to present a comprehensive, mathematical approach for handling uncertainty in the site assessment process. This paper does not present recommended values for various uncertainty sources. Rather, it offers an analytical method for categorizing uncertainty sources in all steps of the site assessment process, and then combining the uncertainty sources to yield an overall estimate of the *AEP* uncertainty. Often, the calculation of the uncertainty in the site assessment process is oversimplified or based on faulty assumptions. This works aims to provide a method for site assessment uncertainty analysis founded on a sound mathematical basis. The methods for combining uncertainty sources in a given calculation rely heavily on the calculation itself. Thus, when describing the determination of the wind resource uncertainty or the *AEP* uncertainty, the actual method for determining the wind resource or the *AEP* must be explained as well.

A. The Weibull Distribution

This paper utilizes the Weibull distribution as an approximation of the wind speed distribution. The Weibull distribution is commonly used to model wind speed distributions, and often provides a good approximation. It relies on two parameters: the scale factor c and the shape factor k. The Weibull probability density function, where U is the wind speed, is shown in Eq. (1).

$$p(U) = (k/c)(U/c)^{k-1}\exp\left(-(U/c)^k\right) \tag{1}$$

A statistical model approximation to the wind speed distribution, as opposed to simply using the measured time series or the frequency distribution of the measured data, is useful for several reasons. First, a statistical model allows for the quality of the wind resource to be quantified by the parameters of the model. In the case of the Weibull distribution, the quality of the wind resource is easily summarized by the values of c and k. Second, a statistical model is extremely useful for handling uncertainty both in wind resource assessment and in *AEP* estimation. Many of the statistical techniques presented in this paper rely on the ability to express the wind speed distribution in a functional form. On the other hand, a statistical model approximation can introduce error into the process, especially when the model does not provide a good fit to the data. The uncertainty associated with the quality of the Weibull fit is not considered in this paper.

By using a Weibull distribution to represent the wind speed distribution, the uncertainty in the wind resource can be expressed as uncertainty in the values of c and k. The scale factor c is nearly proportional to the mean wind speed, and it is reasonable to assume that the percentage uncertainty in the mean wind speed is equal to the percentage uncertainty in c. This is an especially useful assumption since it is often easier to conceptualize and estimate uncertainty in the mean wind speed rather than c.

II. Review of Error Sources and Uncertainty Analysis

All measurements, no matter how carefully done, are subject to errors, which results in a measured value differing from the 'true' value. The amount by which they differ (the size of the error) is unknown, and so all measurements are subject to some uncertainty. In this paper, the term 'uncertainty' will be used as a general measure of the size of the error. The error in a measurement is comprised of two components: the random error and the systematic error.[1]

A. Random Error and Uncertainty

Random error is produced by variability in the quantity being measured or in the measurement procedure. For example, when measuring the duration of an event using a stopwatch, random error may arise when multiple measurements are made. The reaction time of the stopwatch operator can vary, causing each measurement to differ. The standard deviation of the measurements is a measure of the uncertainty of a single measurement due to random error. These errors are often assumed to have normal distributions about the true value.

Often, the mean of the measurements is the quantity of interest. The uncertainty of the mean of the measurements is not equal to the standard deviation of the measurements. Rather, the uncertainty of the mean of the measurements, $\delta\bar{x}$, is equal to the standard deviation of the measurements, σ_x, divided by the square root of the number of measurements, N, assuming the measurements are independent. This relation is shown in Eq. (2).[1]

$$\delta\bar{x} = \sigma_x / \sqrt{N} \tag{2}$$

The uncertainty in the estimate of the mean decreases as the number of measurements increases. Furthermore, this uncertainty will be normally distributed for a large N, even if the distribution of the measurements is not normal. Sources of random error will be categorized as Type A uncertainty.

B. Systematic Error and Unknown Bias Uncertainty

Systematic errors, or biases, are constant over a set of identical measurements. These errors are often due to an error in a calibration constant. An example of a systematic error is a stopwatch that runs slowly. Any measure of the duration of an event with that stopwatch will result in a value that is smaller than the true value. Systematic errors cannot be revealed by repeated measurements because they are constant over the set of measurements, assuming the same instruments are used.

Whenever a measurement is performed, effort should be made to identify the systematic errors, and to either remove the source of the errors or to adjust the measurements by the value of the bias. For example, if one knew that the stopwatch ran slowly by a certain amount, all measurements could be scaled to correct for this known bias. This scaled value is then the estimate of the true value of the measurement. Once the bias has been accounted for by scaling the measurements, the uncertainty is expressed as a value around the scaled value. An estimate of the bias of an instrument often requires comparison to an unbiased instrument or comparison to measurements from multiple instruments.

The issue is complicated when the bias in a measurement is unknown. Again using the stopwatch example, if one does not know how slowly or quickly a stopwatch is running, this produces additional uncertainty in the measurement. However, the bias, while unknown, is constant across every measurement, and so it does not behave like random error, which varies with each measurement and so can be calculated from the measured data. The uncertainty due to unknown systematic errors has typically been estimated based on experience.[2] Furthermore, this uncertainty does not necessarily have to be characterized by a normal distribution, and therefore measured by the standard deviation.

This paper proposes to resolve the issue as follows. While any particular instrument can be subject to an unknown bias, the bias of a collection of all of those instruments is assumed to be normally distributed with a mean value of zero. For example, the mean bias of all the stopwatches from a certain model is assumed to be zero, and variability in the bias of the instruments is assumed to be normally distributed about zero. When one randomly selects a stopwatch from the set of all stopwatches, the uncertainty due to an unknown bias is equal to the standard deviation of the biases of the set of stopwatches. Thus, there is a random component to the unknown bias, even if it is constant for a single instrument. Therefore, the unknown bias uncertainty will be characterized by a normal distribution, and so the standard deviation is the measurement of the uncertainty. The standard deviation of the unknown bias can be measured if multiple instruments are used simultaneously. However, when only a single instrument is used, then the uncertainty has to be approximated. The uncertainty due to unknown bias will be categorized as Type B uncertainty.

C. Combination of Uncertainties

This paper assumes that all Type A and Type B uncertainties are independent and normally distributed.[2,3] Uncertainties will be characterized by the fractional standard uncertainty. The fractional standard uncertainty is a percentage uncertainty, and is calculated as the uncertainty of the measurements of a parameter divided by the absolute value of the expected value of the parameter. It is generally more convenient and intuitive to use the fractional standard uncertainty, since it is non-dimensional. In contrast, the absolute standard uncertainty of a quantity is simply the uncertainty of the measurements of the parameter and so it has units. Throughout this paper, a superscript * is used to denote absolute uncertainties. Fractional uncertainties do not have a * superscript.

Sometimes, specific sources of Type B uncertainty are more easily characterized by an uncertainty limit. That is, the estimated distribution may be rectangular, with equal probability of any error over some range, +/- R. In these cases, the standard uncertainty, δx, is equal to R divided by the square root of three, shown in Eq. (3).[3]

$$\delta x = R / \sqrt{3} \tag{3}$$

When multiple uncertain quantities are used to calculate some parameter, the uncertainties in the component quantities combine to yield a total uncertainty in the parameter. For a parameter f, that is a function of several variables, $f=f(x_1,\ldots, x_n)$, the uncertainties of the variables, $\delta x_1^*,\ldots, \delta x_n^*$, are combined to yield an overall uncertainty, δf^*. δf^* is calculated using Eq. (4), as long as the uncertainties are independent. All uncertainties in Eq. (4) are absolute uncertainties, and so they can have units.

$$\delta f^* = \sqrt{\left(\frac{\partial f}{\partial x_1}\delta x_1^*\right)^2 + \ldots + \left(\frac{\partial f}{\partial x_n}\delta x_n^*\right)^2} \tag{4}$$

Equation (4) is referred to as the "root-sum-square" (RSS) technique, and it the standard method for combining independent uncertainties.[1] Equation (4) can be non-dimensionalized so that the uncertainties are expressed as fractional uncertainties. The non-dimensional form of Eq. (4) is shown in Eq. (5). In Eq. (5), δf and $\delta x_1,\ldots, \delta x_n$ are now fractional uncertainties. The partial derivatives and the fractions, which multiply the fractional uncertainties, are referred to as "sensitivity factors," since they measure how sensitive changes in f are to changes in the variables. The sensitivity factors may be positive or negative in order to indicate if a change in the individual variable causes an increase or a decrease in f. The sign is not particularly important though, since the terms are then squared. The sensitivity factors are also non-dimensional.

$$\delta f = \sqrt{\left(\frac{\partial f}{\partial x_1}\frac{x_1}{f}\delta x_1\right)^2 + \ldots + \left(\frac{\partial f}{\partial x_n}\frac{x_n}{f}\delta x_n\right)^2} \tag{5}$$

Type A and Type B uncertainties can be combined together using Eq. (5) as long as they are each expressed using the standard deviation as a measure of uncertainty. It must be emphasized that there is no definitive method for combining Type A errors and Type B errors. The RSS method is likely the best available option, assuming that the individual uncertainty sources are independent.[1,2] Also, by the central limit theorem the distribution of the total uncertainty will tend toward a normal distribution, regardless of the distribution of the individual sources of uncertainty. In this paper, all sources of uncertainty are assumed to be normally distributed, so the distribution of the total uncertainty will also be normal.

III. Wind Resource Uncertainty

Wind resource assessment is the first major step in the wind energy site assessment process. It consists of using measured wind speed data to estimate the long-term hub height wind resource at the turbine location. Because the wind resource varies from year to year, an estimate of the long-term value is critical to accurately estimate energy production. Also, wind turbine power output depends on the wind speed at the turbine hub height, and so an estimate of the hub height wind resource is necessary for accurate estimations of *AEP*. When the Weibull distribution is used to characterize the wind resource, the goal of wind resource assessment is to estimate the long-term hub height values of the Weibull parameters, c_{LT_HUB} and k_{LT_HUB}. The values of these parameters can then be used in the estimate of *AEP*.

In general, wind resource assessment is a time consuming process, subject to a great deal uncertainty. This section describes the four categories of uncertainty sources that arise in wind resource assessment. Within each category of uncertainty, there are several individual uncertainty sources. A brief description of each category, including the individual component uncertainty sources is provided. There are a total of fourteen individual component uncertainty sources identified in this paper that contribute to the wind resource uncertainty. Next, a method is presented for estimating c_{LT_HUB} and k_{LT_HUB} from measured wind speed data. This process for estimating the wind resource informs the process for estimating the wind resource uncertainty. Finally, a method for estimating the wind resource uncertainty is presented. The method uses the error analysis techniques outlined above. Specifically, sensitivity factors are calculated for the various categories of uncertainty. Because the Weibull distribution is used to characterize the wind resource, the wind resource uncertainty can be expressed as uncertainty in the values of c, and therefore the mean wind speed, and k.

A. Categories of Wind Resource Uncertainty

The causes of uncertainty in wind resource assessment can be subdivided into four categories: Wind Speed Measurement Uncertainty, Long-term Resource Estimation Uncertainty, Wind Resource Variability Uncertainty, and Site Assessment Uncertainty. These four categories will be labeled with subscripts ‘M’, ‘LT’, ‘V’, and ‘SA.’ Each category is now discussed briefly, and the component uncertainty sources within each category are listed.

1. Wind Speed Measurement Uncertainty

These uncertainty sources arise when measuring the actual wind speed at a site. The wind speed at a site is usually measured by taking 10-minute averages of the wind speed, sampled at approximately 1 Hz. Wind data at a site are then presented as a time series of these 10-minute averages. The mean measured wind speed at a site is the mean of all the values of the measured wind speed. Likewise, the measured wind speed can be used to estimate the Weibull parameters. The measured mean wind speed and Weibull parameters can be labeled $\bar{U}_M$, c_M, and k_M, respectively.

Several factors can contribute to errors in the measurement of the wind speed, and therefore in the determination of the $\bar{U}_M$, c_M, and k_M. These factors fall into the category of wind speed measurement uncertainty. They are:

1. Anemometer Uncertainty I - Calibration Uncertainty
2. Anemometer Uncertainty II - Dynamic Overspeeding
3. Anemometer Uncertainty III - Vertical Flow Effects
4. Anemometer Uncertainty IV - Vertical Turbulence Effects
5. Tower Effects
6. Boom and Mounting Effects
7. Data Reduction Accuracy

2. Long-term Resource Estimation Uncertainty

These uncertainty sources arise when the measured wind resource data are used to estimate the long-term wind resource at a site. While wind resource measurement typically lasts for 1 year, the measured resource during this particular year may not be representative of the actual long-term resource at the site. The long-term resource is characterized by the mean wind speed and wind speed distribution that exists at a site over a very long period of time. Typically, twenty years is assumed to be a long enough time period to characterize the long-term wind resource. Since a twenty-year measurement campaign is far too long for practical purposes, the long-term resource must be estimated from the measured data. The measured data, along with long-term data from a nearby site (the "reference site"), is generally used in a process called Measure-Correlate-Predict (MCP) to estimate the long-term wind resource at a site. The long-term mean wind speed and Weibull parameters can be labeled $\bar{U}_{LT}$, c_{LT}, and k_{LT}, respectively.

MCP introduces uncertainty into the estimate of the long-term resource at the site. Furthermore, the estimation of k_{LT} in the MCP step contributes an additional uncertainty because different parameter estimation models yield different values for k_{LT}. Finally, global climate change may also cause uncertainty in the estimate. Thus, additional uncertainty arises when the measured data are used to estimate the long-term wind resource at a site. The three uncertainty sources that arise in long-term resource estimation are:

8. MCP Correlation Uncertainty
9. Weibull Parameter Estimation Uncertainty
10. Changes in the Long-term Average

3. Wind Resource Variability Uncertainty

When the wind resource at a site is evaluated, a finite number of years of data are used to estimate the long-term mean wind speed and Weibull parameters. Essentially, a sample of yearly mean wind speeds is used to estimate the long-term wind resource. Thus, the potential for random error exists, and it causes uncertainty in two ways. First, the reference site data used in the MCP step to estimate the long-term parameters might not in fact be representative of the true long-term values. Since random error decreases as the number of samples increases, the longer the reference site data set used in the MCP step to estimate the long-term parameters, the less uncertainty there will be in these estimates. Second, the actual wind resource over the lifetime of a turbine may not be the same as the true long-term wind resource, which produces additional uncertainty. These two uncertainties are therefore Type A uncertainties, and they are:

11. Inter-Annual Variability Uncertainty
12. Uncertainty over Turbine Lifetime

4. Site Assessment Uncertainty

Wind speed measurements usually take place at heights significantly lower than the hub height of a typical modern wind turbine. Because wind speeds typically increase with height, a wind shear model is used to extrapolate the estimated long-term wind resource to the hub height. The wind shear model is created using the measured wind speed data. The use of a wind shear extrapolation introduces uncertainty. Furthermore, the tower used to measure the wind speed is often not at the exact location of the wind turbine(s). Topographic effects can cause the wind speeds at separate locations at a site to differ. Thus, site assessment uncertainties arise when estimating the long-term, hub height mean wind speed and Weibull parameters at the probable turbine location, $\bar{U}_{LT\_HUB}$, c_{LT_HUB}, and k_{LT_HUB}, from the long-term mean wind speed at the measurement height and the measurement location, $\bar{U}_{LT}$, c_{LT}, and k_{LT}. The two factors contributing to this uncertainty are:

13. Topographic Effects
14. Wind Shear Model Uncertainty

B. Estimation of the Wind Resource

Once the wind speed measurements at the site are completed and the fourteen sources of wind resource uncertainty have been estimated, an estimate can be made of the long-term wind resource and the associated uncertainty in this estimate. This section describes the process that should be used for estimating the long-term hub height mean wind speed and Weibull parameters, $\bar{U}_{LT\_HUB}$, c_{LT_HUB}, and k_{LT_HUB}. The next section discusses the associated uncertainty. Again, it is important to review the process for estimating the wind resource because it helps frame the process for estimating the uncertainty. The steps that should be used to determine the long-term, hub height wind resource are listed below.

1. Correct for any known bias in the measured wind speed. The anemometer vertical sensitivity, the anemometer overspeeding, the anemometer vertical turbulence effects, and the tower effects may introduce a bias into the measurement of the wind speed. The measured wind speed data should be scaled to remove these biases.

2. Use the measured wind speed data to determine the appropriate shear parameter. If the power law is used, then solve for α. If the log law is used, then solve for z_0, assuming a displacement height of zero.

3. Use the measured wind speed data and data from a long-term reference site in an MCP algorithm. The outputs of the MCP algorithm are estimates of the long-term mean wind speed and long-term Weibull parameters at the measurement height and location, $\bar{U}_{LT}$, c_{LT}, and k_{LT}. The Weibull parameters can be estimated in a variety of ways, including empirical methods and maximum likelihood estimation.

4. Assume that k does not change with height, and therefore $k_{LT} = k_{LT\_HUB}$. This assumption is based on a review of shear models for k.[4,5] While various models exist for extrapolating k with height, the predictions are highly variable, with little predictive value. Furthermore, for reasonably high measurement heights (~50 m) and modern turbine hub heights, the models predict small increases in k. Overall, it appears reasonable to assume that k is unchanged with height.

5. Apply the chosen shear model to $\bar{U}_{LT}$ and c_{LT}, the outputs of the MCP step. This can be done as follows. The Weibull parameters relate to the mean wind speed, $\bar{U}$, according to Eq. (6), where Γ is the gamma function.[6]

$$c = \bar{U} / \Gamma\left(1 + 1/k\right) \tag{6}$$

The mean wind speed and c are proportional to each other, and the constant of proportionality is only a function of k. Therefore, if k remains constant with height, then the constant of proportionality is also constant. The result is that one can apply the same shear law to c_{LT} that one applies to $\bar{U}_{LT}$. For example, if one were utilizing the power law, then after determining α in Step 2, Eq. (7) could be used to calculate $\bar{U}_{LT\_HUB}$ and c_{LT_HUB}. h_3 is the probable hub height, and h_2 is the highest measurement height.

$$\bar{U}_{LT\_HUB} = \bar{U}_{LT}\left(h_3 / h_2\right)^{\alpha} \quad , \quad c_{LT\_HUB} = c_{LT}\left(h_3 / h_2\right)^{\alpha} \tag{7}$$

6. Correct for any known bias in the shear extrapolation and topographic correction. These processes can introduce bias, and so the estimated values of $\bar{U}_{LT\_HUB}$ and c_{LT_HUB} should be scaled by these biases.

C. Wind Resource Uncertainty

Once the long-term wind resource has been determined, the uncertainty of the long-term wind resource can then be estimated. This section describes the estimation of the uncertainty of $\bar{U}_{LT\_HUB}$, c_{LT_HUB}, and k_{LT_HUB}. The estimates of the individual uncertainty values for the mean wind speed are labeled δU_i, and for the shape factor they are labeled δk_i. It is assumed that the uncertainty in the mean wind speed equals the uncertainty in c, so $\delta U_i = \delta c_i$ in all cases. The steps to determine these uncertainties are:

1. Use the "root-sum-square" (RSS) method to combine the uncertainty values within each category of uncertainty. This is shown in Eq. (8). The subscripts 'M', 'LT', 'V', and 'SA' correspond to uncertainty in the four categories of uncertainty. Within each category of uncertainty, the sensitivity factors are equal to one. That is, for a given category of uncertainty (e.g. Category I: Measurement Uncertainty), the sensitivity factor of each uncertainty is one.

$$\begin{aligned} I: \quad \delta U_M &= \sqrt{(\delta U_1)^2 + (\delta U_2)^2 + (\delta U_3)^2 + (\delta U_4) + (\delta U_5)^2 + (\delta U_6)^2 + (\delta U_7)^2} \\ II: \quad \delta U_{LT} &= \sqrt{(\delta U_8)^2 + (\delta U_9)^2 + (\delta U_{10})^2} \\ III: \quad \delta U_V &= \sqrt{(\delta U_{11})^2 + (\delta U_{12})^2} \\ IV: \quad \delta U_{SA} &= \sqrt{(\delta U_{13})^2 + (\delta U_{14})^2} \end{aligned} \tag{8}$$

2. The general equation to determine δU, which is the long-term hub height mean wind speed uncertainty, is derived from Eq. (5). The result is shown in Eq. (9). The sensitivity factors for each category of uncertainty are SF_M, SF_{LT}, SF_V, and SF_{SA}.

$$\delta U = \sqrt{(SF_M \cdot \delta U_M)^2 + (SF_{LT} \cdot \delta U_{LT})^2 + (SF_V \cdot \delta U_V)^2 + (SF_{SA} \cdot \delta U_{SA})^2} \tag{9}$$

3. Determine the sensitivity factors for each category of uncertainty. The sensitivity factor for the wind resource variability uncertainty and the site assessment uncertainty are both equal to one. Thus, $SF_V = SF_{SA} = 1$.

If a linear model is used in the MCP step, then the sensitivity factor for the long-term resource estimation, SF_{LT}, is also equal to one. RERL uses a linear MCP model, dubbed the "Variance Ratio" method, which has been shown to perform very well relative to other models.[7] From an uncertainty perspective, using a linear model helps simplify the calculation of the overall wind resource uncertainty.

The sensitivity factor for the measurement uncertainty, SF_M, is not equal to one. SF_M is not equal to one because the measured wind speed is used to calculate the shear parameter, and the shear parameter is then used to estimate $\overline{U}_{LT\_HUB}$ and c_{LT_HUB}. The result is that error in the measurement of the wind speed causes error in the shear parameter calculation, which then causes additional error in the estimate of $\overline{U}_{LT\_HUB}$ and c_{LT_HUB}.[8] Thus, the contribution of measurement uncertainty to the total uncertainty is magnified due to shear extrapolation, and so the sensitivity factor for the measurement uncertainty is greater than one. It is important to emphasize that this effect is not due to any error in the wind shear model. Rather, it is a mathematical byproduct of using uncertain data to determine an extrapolation parameter.

SF_M can be calculated as follows. h_1, h_2, and h_3 are the heights of the lower measurement anemometer, the higher measurement anemometer, and the hub height, respectively. $\overline{U}_{M1}$ and $\overline{U}_{M2}$ are the measured mean wind speeds at the lower and upper anemometer, respectively. When the power law is used, the measured data can be used to calculate the shear parameter, α, using Eq. (10).

$$\alpha = \ln\left(\overline{U}_{M2} / \overline{U}_{M1}\right) / \ln\left(h_2 / h_1\right) \tag{10}$$

The predicted mean wind speed, $\overline{U}_{HUB}$, at height h_3 can then be calculated using Eq. (11).

$$\overline{U}_{HUB} = \overline{U}_{M2}\left(h_3 / h_2\right)^{\ln\left(\overline{U}_{M2}/\overline{U}_{M1}\right)/\ln\left(h_2/h_1\right)} \tag{11}$$

If it is assumed that the uncertainties in the mean wind speeds are normally distributed, and there is no uncertainty in the three heights, then the uncertainties can be related using Eq. (12). The uncertainties in Eq. (12) are absolute uncertainties.

$$\delta\overline{U}^*_{HUB} = \sqrt{\left(\frac{\partial\overline{U}_{HUB}}{\partial\overline{U}_{M1}}\delta\overline{U}^*_{M1}\right)^2 + \left(\frac{\partial\overline{U}_{HUB}}{\partial\overline{U}_{M2}}\delta\overline{U}^*_{M2}\right)^2} \tag{12}$$

Next, it is assumed that the fractional standard uncertainty of $\overline{U}_{M1}$ and $\overline{U}_{M2}$ ($\delta\overline{U}_{M1}$ and $\delta\overline{U}_{M2}$), are both equal to the fractional standard measurement uncertainty, δU_M, as shown in Eq. (13). That is, it is assumed that the measurement uncertainties at both heights are identical.

$$\delta U_M = \delta\overline{U}_{M1} = \delta\overline{U}_{M2} = \delta\overline{U}^*_{M1} / \overline{U}_{M1} = \delta\overline{U}^*_{M2} / \overline{U}_{M2} \tag{13}$$

Finally, after substituting Eq. (13) into Eq. (12), and after some algebraic manipulation, the ratio of the fractional uncertainty in the predicted mean wind at height h_3, $\delta\overline{U}_{HUB}$, to the fractional standard measurement uncertainty, δU_M, can be written using Eq. (14). This ratio is also equal to the sensitivity factor for the measurement uncertainty, SF_M.

$$SF_M = \frac{\delta\overline{U}_{HUB}}{\delta U_M} = \sqrt{\left(\frac{\partial\overline{U}_{HUB}}{\partial\overline{U}_{M1}}\frac{\overline{U}_{M1}}{\overline{U}_{M2}}\right)^2 + \left(\frac{\partial\overline{U}_{HUB}}{\partial\overline{U}_{M2}}\right)^2} \cdot \left(\frac{\overline{U}_{M2}}{\overline{U}_{HUB}}\right) \tag{14}$$

The partial derivatives can be calculated using Eq. (11). Likewise, the ratios $\overline{U}_{M1}/\overline{U}_{M2}$ and $\overline{U}_{M2}/\overline{U}_{HUB}$ can be calculated using Eq. (11). When these calculations are substituted into Eq. (14), SF_M can be written as an analytic function of only the three measurement heights. The final result for SF_M is shown in Eq. (15).

$$SF_M = \sqrt{\frac{2\left(\ln\left(\frac{h_3}{h_2}\right)\right)^2 + \left(\ln\left(\frac{h_2}{h_1}\right)\right)^2 + 2\ln\left(\frac{h_2}{h_1}\right)\ln\left(\frac{h_3}{h_2}\right)}{\left(\ln\left(\frac{h_2}{h_1}\right)\right)^2}} \quad (15)$$

3. Calculate the overall uncertainty in the long-term hub height mean wind speed, δU. This is accomplished using Eq. (9). Because $SF_V = SF_{SA} = SF_{LT} = 1$, the equation for δU can be written as shown in Eq. (16).

$$\delta U = \sqrt{\left(SF_M \cdot \delta U_M\right)^2 + \left(\delta U_{LT}\right)^2 + \left(\delta U_V\right)^2 + \left(\delta U_{SA}\right)^2} \quad (16)$$

4. Determine δc. As stated above, the uncertainty in $\bar{U}_{LT\_HUB}$ and c_{LT_HUB} are equal to each other, so $\delta U = \delta c$.

IV. Wind Turbine Power Production and Uncertainty

The previous section focused on the process of evaluating the wind resource at a site and the associated uncertainty. Like the wind resource, the determination of the power curve and power production of a wind turbine is also potentially subject to error, which then causes uncertainty in the estimate of *AEP*.

A. Wind Turbine Power Production Uncertainty Sources

Three sources of power production uncertainty were identified. A detailed discussion of the causes and sizes of these uncertainty sources will not be presented. Rather, the three uncertainty sources are listed and a brief discussion is provided. The three sources of power production uncertainty are labeled δP_1, δP_2, and δP_3. They are:

1. Wind Turbine Specimen Variation
2. Wind Turbine Power Curve Uncertainty
3. Air Density Uncertainty

The power curve uncertainty is typically significantly larger than the other two uncertainties. When power curves for wind turbines are measured by the manufacturer, several factors contribute to the uncertainty in this ‘measured power curve.’ The primary factor is uncertainty in the wind speed to which the turbine is responding because the uncertainty in the actual power being produced is quite small. While the wind speed at the hub height is known to a fairly high accuracy, the effects of turbulence and shear across the rotor face are not taken into account, and consequently the mean wind speed averaged over the rotor face is uncertain.

This issue is exacerbated when a turbine is placed at a particular site because the power curve of a wind turbine is site dependent, and not solely a function of the hub height wind speed. The turbulence, air density, and shear characteristics of a site will affect the power curve of a turbine, with the result that a turbine at a specific site could produce either more or less power than the power curve indicates at a given wind speed. The measured power curve specifically corresponds to a site that meets the IEC standards, which require a flat site with very low turbulence.[3] Thus, a site-specific power curve is needed to estimate energy production at a site, especially when the terrain is complex. The manufacturer generally determines this site-specific power curve.

B. Wind Turbine Power Production Uncertainty

The overall power production uncertainty can be calculated using the general equation given in Eq. (5). The three uncertainty sources are independent, and the sensitivity factor for each is one. Thus, the overall power production uncertainty, δP, is shown in Eq. (17).

$$\delta P = \sqrt{\left(\delta P_1\right)^2 + \left(\delta P_2\right)^2 + \left(\delta P_3\right)^2} \quad (17)$$

V. Wind Turbine Energy Production Losses and Uncertainty

This section describes the factors that contribute to uncertainty in the estimate of energy production. These factors are distinct from those that related to uncertainty in the instantaneous power output. The factors discussed in this section are defined relative to the estimate of the energy production, and not the power output. They are referred to as the "energy loss factors."

A. Energy Loss Factors

Three energy loss factors have been identified. Each of these factors reduces the energy production of a wind turbine or wind farm. They are:

1. Availability Losses
2. Fouling and Icing Losses
3. Array Losses

These three energy loss factors are labeled ELF_{AV}, ELF_{FOUL}, and ELF_{ARRAY}. Each energy loss factor is defined as the ratio of the actual energy produced divided by the ideal energy production if there were no losses. Thus, ELF_{AV} is simply equal to the actual expected energy production of the wind turbine or wind farm, divided by the hypothetical energy production if there was no maintenance of the turbine(s) or down time, whether scheduled or unscheduled. The other two energy loss factors have equivalent definitions. The total reduction to the wind turbine or wind farm energy loss is simply the product of the three energy loss factors. Thus, the overall energy loss factor, ELF, is shown in Eq. (18).

$$ELF = ELF_{AV} \cdot ELF_{FOUL} \cdot ELF_{ARRAY} \tag{18}$$

The energy loss factors are independent and a normal distribution is assumed for each. The justification of this assumption is discussed next.

B. Justification of Normally Distributed Energy Loss Factors

The assumption of a normal distribution must now be justified. The energy loss factors, by definition, have a range between 0% and 100%. A normal distribution is defined between [-∞,∞]. This is a clear contradiction because a normally distributed energy loss factor implies the possibility for a value less than zero or greater than one. Despite this contradiction, normally distributed energy loss factors will be used, and there is a sound mathematical basis for their use.

The yearly availability of a wind farm is not a normally distributed quantity about its expected value. This can be seen clearly in the histogram in Fig. 1. The data in Fig. 1 show the number of occurrences of yearly availability values for 25 different wind farms, with a total of 104 wind farm-years of operation. The mean availability is approximately 94%, and the distribution is clearly asymmetrical, with an upper limit of 100%. These data were compiled from a variety of North American wind farms.[9]

The data in Fig. 1 can be fit with a Weibull distribution, for example. The choice of a Weibull distribution is fairly arbitrary, though it does provide a good fit to the empirical data. The Weibull approximation to the data is also shown in Fig. 1. The shape factor has a value of k=1.5.

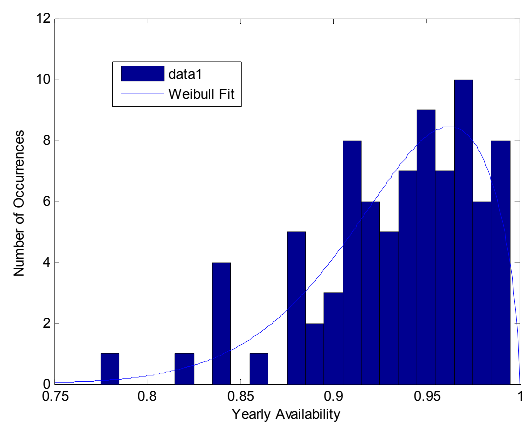

Figure 1. Empirical Availability Data and Weibull Fit

The yearly availability is not the quantity used in energy production estimates, however. When estimating energy production, the average availability over the approximately twenty-year lifetime of the project is used in the estimate. Thus, the quantity of interest is the lifetime availability of the wind farm, or the average of the twenty yearly availability values. The distribution of the lifetime availability can be determined using a 'Monte Carlo' simulation, assuming the yearly availability in any given year is independent of other years. The simulation proceeded as follows:

1. Using a Matlab function, twenty values were randomly sampled from a Weibull distribution with an expected value of 0.94 and a k=1.5, which is the Weibull fit to the empirical availability data. These twenty values represent a random set of wind farm yearly availability values.

2. These twenty yearly availability values were averaged to get the lifetime availability.

3. This process was repeated 100,000 times.

The distribution of the lifetime availability values can then be plotted in a histogram. This is shown in Fig. 2. A normal distribution can be fit to the data, and is shown in Fig. 2 as well. The mean is 0.94 and the standard deviation is 0.009.

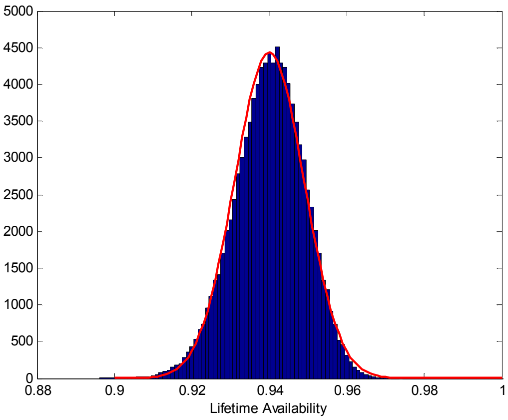

Figure 2. Distribution of Lifetime Availability Values

Thus, while the distribution of yearly availability values can be approximated by a Weibull distribution, the distribution of lifetime availability values is very closely approximated by a normal distribution. This result should not be surprising, and it follows directly from the central limit theorem. This result makes intuitive sense as well. The yearly availability distribution from Fig. 1 indicates that there is approximately a 10% chance of getting an availability value less than 0.90 in a single year. However, the chance of getting availability values less than 0.90 over the lifetime of the project is extremely small, and this is reflected in Fig. 2. Over a twenty-year period, it is unlikely that the average availability will be much different than the expected value.

This result can be applied to all three energy loss factors. Like the availability, the yearly distributions of ELF_{FOUL} and ELF_{ARRAY} cannot be normally distributed. Instead, they most likely follow a similar asymmetrical distribution, with an upper limit of one. However, the distribution of the twenty-year average will once again follow a normal distribution, due to the central limit theorem. Thus, all three energy loss factors will be characterized by normal distributions.

There is still the possibility for values greater than one when normally distributed energy loss factors are assumed, and so a contradiction remains. In general, however, the probability of a value greater than one will be so small that it is completely negligible. In the availability example, the probability of lifetime availability greater than one is 0.000000002%. Clearly the contradiction can be ignored in this case.

C. Energy Loss Factors and Uncertainty

The energy loss factors are independent and normally distributed, and the overall energy loss factor uncertainty, δELF, can be calculated using Eq. (5), as long as the individual energy loss factor uncertainties, δELF_{AV}, $\delta ELF_{\mathrm{FOUL}}$, and $\delta ELF_{\mathrm{ARRAY}}$, are expressed as fractional standard uncertainties. The sensitivity factor for each energy loss factor is one, since the overall energy loss factor is simply the product of the three individual energy loss factors. The resulting equation for δELF is given in Eq. (19).

$$\delta ELF = \sqrt{\left(\delta ELF_{AV}\right)^2 + \left(\delta ELF_{FOUL}\right)^2 + \left(\delta ELF_{ARRAY}\right)^2} \quad (19)$$

VI. Energy Production and Uncertainty

Once the wind resource at a site has been determined, it is combined with a selected power curve and the energy loss factors to yield an estimate of the energy production of the wind turbine or wind farm. The uncertainty in the wind resource, the power production, and the energy loss factors contribute to an overall uncertainty in the energy production.

Often, it is more convenient to use the "Capacity Factor," as a measure of energy production. The capacity factor, CF, of a wind turbine is simply equal to the average turbine power output, $\overline{P}_{\mathrm{W}}$, divided by the rated turbine power output, P_{R}, and so it is a non-dimensional quantity. The relationships between CF, AEP, and average power are shown in Eq. (20). CF will be used exclusively as a measure of energy production for the rest of this section.

$$CF = \overline{P_W} / P_R, \quad AEP = \overline{P_W} \cdot \left(8766 \quad hours\right) \quad (20)$$

This section will review both how the wind resource, power curve, and energy loss factors are combined to estimate *CF*, and how the uncertainties of each of these terms are combined to yield the uncertainty of *CF*. The methods described in this section can be applied to either a single wind turbine or a wind farm. The total wind farm energy production is simply the sum of the energy productions of each turbine, and the wind farm capacity factor is simply the average of all the wind turbine capacity factors. The effect of wake losses on individual turbines is encompassed in the array loss energy loss factor ELF_{ARRAY}, which is a measure of the total energy loss of the wind farm due to wake losses. ELF_{ARRAY} is typically calculated using a wake model.

A. Capacity Factor Estimation

The capacity factor, *CF*, is a function of the estimate of the long-term hub height Weibull parameters, which will simply be labeled *c* and *k* in this section, along with a wind turbine power curve, P_{W}, and the overall energy loss factor, *ELF*, as shown in Eq. (21).

$$CF = CF\left(c, k, P_W, ELF\right) \tag{21}$$

The actual functional dependence can be found by integrating the product of the wind speed probability distribution and the power curve over all values of wind speed, *U*, and then multiplying by the overall energy loss factor, and dividing by the rated power. The wind speed probability distribution is the Weibull distribution, given in Eq. (1), with the long-term hub height values of *c* and *k* used as the Weibull parameters. This is shown in Eq. (22).[6]

$$CF = \overline{P_W} / P_R = \left(ELF / P_R\right) \cdot \int_0^\infty P_W(U)\left(k/c\right)\left(U/c\right)^{k-1} \exp\left[-\left(U/c\right)^k\right] dU \tag{22}$$

The integral in Eq. (22) cannot be performed analytically. Instead, it must be approximated numerically. A trapezoid method or Simpson’s method can be easily implemented to perform this integral.

B. Capacity Factor Uncertainty Estimation

While the uncertainties *δc*, *δk*, *δP*, and *δELF* are known, the non-linear dependence of *CF* on *c* and *k* makes the calculation of the total uncertainty of *CF*, *δCF*, fairly complicated. The general equation for the uncertainty in *CF* is given in Eq. (23).

$$\delta CF = \sqrt{\left[\left(\frac{\partial CF}{\partial c}\frac{c}{CF}\right)\delta c\right]^2 + \left[\left(\frac{\partial CF}{\partial k}\frac{k}{CF}\right)\delta k\right]^2 + \delta P^2 + \delta ELF^2} \tag{23}$$

This is derived from the general uncertainty formula for fractional uncertainties, shown in Eq. (5). Thus, *δc*, *δk*, *δP*, and *δELF* are the fractional standard uncertainties. The terms multiplying *δc* and *δk* (the partial derivatives and the fractions inside the parentheses) in Eq. (23) are the sensitivity factors for *c* and *k*. Like the uncertainties, the sensitivity factors are non-dimensional. The sensitivity factors for *c* and *k* will be labeled SF_{c} and SF_{k}, respectively. The sensitivity factors for *δP* and *δELF* are 1.

The calculation of SF_{c} and SF_{k} begins with Eq. (22). The partial derivatives can be taken using the Leibniz integration rule, shown in Eq. (24) and Eq. (25). *p*(*U*) is the Weibull probability density function.

$$SF_c = \frac{\partial CF}{\partial c}\frac{c}{CF} = \frac{1}{CF}\frac{ELF}{P_R}\int_0^\infty P_W(U)\frac{\partial p(U)}{\partial c} c\, dU \tag{24}$$

$$SF_k = \frac{\partial CF}{\partial k}\frac{k}{CF} = \frac{1}{CF}\frac{ELF}{P_R}\int_0^\infty P_W(U)\frac{\partial p(U)}{\partial k} k\, dU \tag{25}$$

After the derivatives are taken and some algebraic manipulation is carried out, the sensitivity factors can be written as shown in Eq. (26) and Eq. (27). The integrals cannot be calculated analytically, but once again numerical integration can be used to estimate the sensitivity factors.

$$SF_c = \frac{1}{CF}\frac{ELF}{P_R}\int_0^\infty P_W(U)k\left(\left(\frac{U}{c}\right)^k - 1\right)p(U)dU \tag{26}$$

$$SF_k = \frac{1}{CF}\frac{ELF}{P_R}\int_0^\infty P_W(U)\left(1 + k\ln\left(\frac{U}{c}\right)\left(1-\left(\frac{U}{c}\right)^k\right)\right)p(U)dU \tag{27}$$

Finally, the total uncertainty of *CF* can be calculated using the general equation given from Eq. (23). Again, δc, δk, δP, and δELF are all fractional uncertainties.

$$\delta CF = \sqrt{(SF_c \cdot \delta c)^2 + (SF_k \cdot \delta k)^2 + \delta P^2 + \delta ELF^2} \tag{28}$$

In sum, *CF* is the estimated value for the capacity factor. It is calculated using the long-term hub height estimates of *c* and *k*, the chosen power curve, and the energy loss factors. The uncertainty of this estimate is δCF, which is the fractional standard uncertainty of *CF*.

C. Summary of Methods to Estimate the Capacity Factor and the Capacity Factor Uncertainty

The estimation of *CF* and δCF is a complicated multi-step process. The steps below summarize the process of estimating these quantities.

1. Estimate the long-term hub height wind resource. This process combines the measured wind resource with an MCP algorithm, a shear extrapolation, and a topographic correction to produce estimates of $c_{\text{LT\_HUB}}$ and $k_{\text{LT\_HUB}}$.
2. Estimate δc and δk. Fourteen different uncertainty sources contribute to the overall uncertainty of the estimate of the long-term Weibull parameters at the hub location of the turbine. The individual uncertainty sources can then be combined using the Eq. (16), once the sensitivity factor for the wind resource measurement uncertainty is calculated in Eq. (15).
3. Choose a wind turbine.
4. Estimate δP. Three uncertainty sources were identified that contribute to the overall uncertainty in the power curve and power production. These uncertainty sources are easily combined using the RSS method to yield an estimate of δP.
5. Estimate the value of the three energy loss factors. The three energy loss factors can then be combined into an overall energy loss factor, *ELF*, using Eq. (18).
6. Estimate the energy loss factor uncertainty, δELF. This can be calculated using Eq. (19).
7. Estimate the capacity factor, *CF*. This is calculated using Eq. (22), and it depends on the long-term hub height Weibull parameters, the chosen power curve, and *ELF*.
8. Estimate SF_C and SF_k. The sensitivity factors for *c* and *k* can be calculated using Eq. (26) and Eq. (27).
9. Estimate δCF. Using the results of Steps 1-8, δCF can be estimated using Eq. (28).

D. Example Calculation of Capacity Factor and Capacity Factor Uncertainty

An example calculation helps to clarify the process because it can be quite complicated. The important assumptions and values in the example are:

1. The long-term hub height Weibull parameters are $c = 9$ m/s and $k = 2.5$.
2. A GE 1.5 MW wind turbine power curve will be used for this example. This power curve is shown in Fig. 3. This is a variable speed, pitch regulated wind turbine.
3. The total energy losses reduce the energy production by 22%, so $ELF = 0.78$.
4. The example uncertainty values of the long-term hub height Weibull parameters, the power

δc	δk	δP	δELF
11.9%	11.8%	10.5%	4.0%

Table 1. Example Uncertainty Values

production, and the energy loss factor are given in Table 1. The basis for these values is outside the scope of this paper.

The result of the example calculation is a value for the capacity factor of CF = 33.1%. This was calculated using Eq. (22) and a numerical integration program. The values for the sensitivity factors for c and k are SF_c = 1.85 and SF_k = 0.07. Using the sensitivity factors and the example uncertainties, the overall capacity factor uncertainty is 24.9%. This is the fractional standard uncertainty. To obtain the absolute standard uncertainty, δCF is multiplied by CF. The result is an absolute standard uncertainty of 0.08. Thus, one could write that the estimate of CF is: CF = 0.331 +/- 0.08.

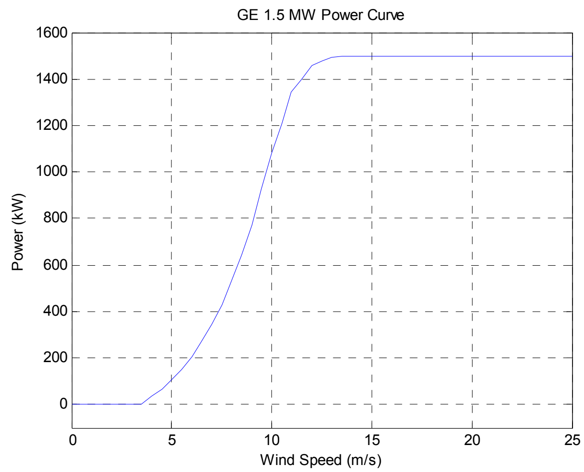

Figure 3. Example Power Curve

E. Example Calculation Discussion

Many important points can be made about the example calculation.

- The value of CF is clearly dependent on the values of both c and k. This dependency is illustrated in Fig. 4, which shows the value of the capacity factor for ranges of values of c and k, using the same power curve and ELF value as the example. Figure 4 clearly shows that the capacity factor increases as c increases. This comes as no surprise, as the value of c is directly proportional to the mean wind speed, which is by far the most important parameter in assessing a site and estimating energy production. Also, CF tends to increase as k increases, except when c is very small. As k increases, the Weibull distribution becomes less spread out and therefore more concentrated about its expected value. At very low values of c, this means that the distribution is concentrated about very low values of the wind speed, and so the turbine is producing very little power, even when it is above the cut-in wind speed. But, for moderate to high values of c, a high k value results in the wind speed being above the cut-in value for a very high percentage of the time, and therefore a high capacity factor.

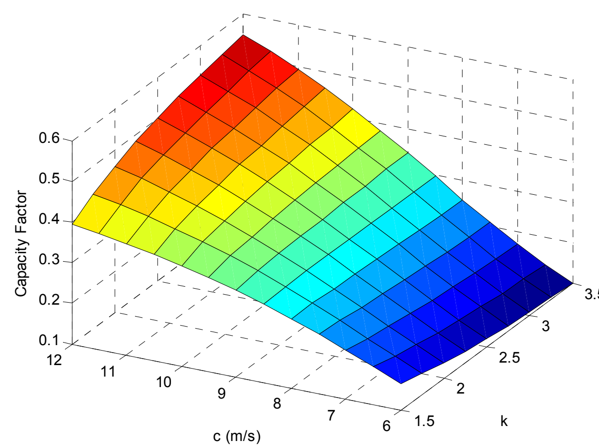

Figure 4. Dependence of *CF* on *c* and *k*

- The magnitude of δCF depends predominantly on the value of δc. In fact, if δk, δP, and δELF were all 0%, the uncertainty in CF would still be 22.1%. This is due to the large magnitude of SF_c. Thus, reducing the uncertainty in c, and so the mean wind speed, provides the best opportunity for reducing the overall capacity factor uncertainty.
- The sensitivity factor for c is 1.85. This is lower than is generally assumed. It is common practice to assume that a percentage increase in c (or the mean wind speed) causes between 2 and 3 times the percentage increase in CF, i.e. a sensitivity factor between 2 and 3.[2] This is an oversimplification, however, and the results indicate that this assumption could lead to large errors. For example, if one had assumed that SF_c = 2.3, then δCF would be 29.7% instead of 24.9%. This could mean the difference between an acceptable and unacceptable risk level for a potential wind energy venture.
- In reality SF_c is highly dependent on the value of c. SF_c decreases as c increases, and increases as k increases. The dependence of SF_c on c and k for this example is shown in Fig 5. The plot indicates that SF_c approximately decreases proportionally to the square of c. As c increases, the wind speed is above the rated wind speed more and more frequently. Therefore any error in the estimation of c affects the value of CF less, since the turbine produces constant power above rated wind speed, regardless of the actual wind speed. As k increases, the Weibull distribution becomes less spread out, resulting in the wind speed being close to the mean value more

frequently. For a given value of *c*, a higher value of *k* results in the wind speed being less than the rated wind speed more often, and therefore the capacity factor is more dependent on the value of *c*, and so SF_c is larger.

- In this example the sensitivity factor for *k* is fairly small relative to the other sensitivity factors, indicating that *CF* has a weak dependence on *k*. For example, a 10% increase in *k* would only change *CF* by 0.7%, for this example. This is not a surprising result because generally the shape of the wind speed distribution is considered much less important for *CF* estimates than the mean wind speed value (or *c*). However, the sensitivity factor of *k* is also highly dependent on the value of both *c* and *k*. Figure 6 shows this dependence for this example.

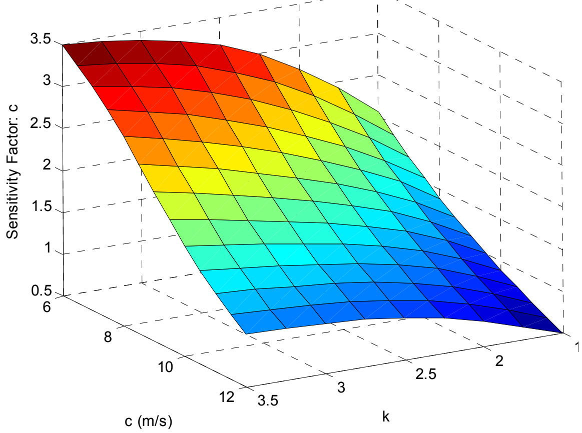

Figure 5. Dependence of SF_c on *c* and *k*

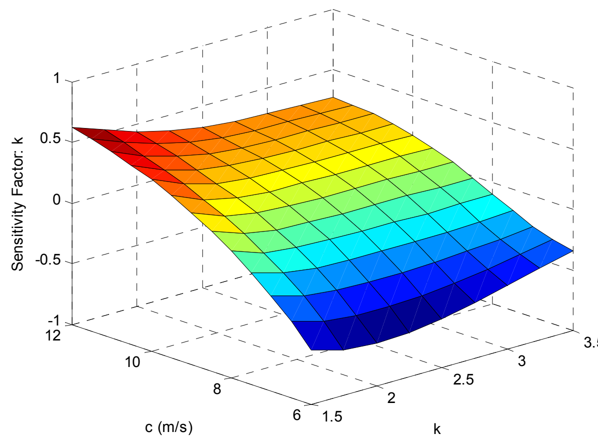

Figure 6. Dependence of SF_k on *c* and *k*

- The value of *k* is often assumed to be approximately 2, although it can vary widely, with values that may be as low as 1.5 or as high as 3.5 at some sites. The value of *k* is not insignificant. For this example, if one assumed that *k* was equal to 2, but in fact it was equal to 1.5, the capacity factor would be calculated as 0.325 or 0.311, respectively. This may seem like a small difference, but it corresponds to a 4% change in *AEP*, which can make the difference between a successful venture and a failure. Thus, assuming a value of *k* may have adverse consequences because it could lead to a significantly incorrect estimation of *CF* (or *AEP*). This point further reinforces the utility of using the two parameter Weibull distribution to approximate the wind speed distribution rather than the one parameter Rayleigh distribution (the Rayleigh distribution is equivalent to the Weibull with a value of k=2). By estimating a value of *k*, and therefore taking the shape of the wind speed probability distribution into account in *AEP* estimation, one can avoid the significant errors that may arise when *k* is assumed to be a certain value.

VII. Conclusion

Wind energy site assessment is a complex multi-step process, with a high degree of uncertainty. This paper seeks to present a comprehensive means for executing the site assessment process, and for understanding and estimating uncertainties in this process. The major conclusions are:

- Mathematically rigorous methods for estimating uncertainty are presented. These methods utilize sensitivity factors to combine independent sources of uncertainty. Specifically, the wind speed measurement uncertainty sensitivity factor and the Weibull parameter sensitivity factors can be calculated explicitly using the methods outlined in this paper. This eliminates the need to assume the values for sensitivity factors, or ignore them altogether. The result is that the uncertainty in the site assessment process can be calculated more accurately.
- From a strategic perspective, wind measurement devices capable of measuring at hub height offer an opportunity to significantly reduce wind resource assessment uncertainty. These devices, such as a LIDAR or SODAR, eliminate errors due to shear model uncertainty, and tower/boom effects. Also, the measurement uncertainty sensitivity factor is 1 when measuring at hub height, which further reduces the wind resource uncertainty. In fact, a LIDAR or SODAR with 5% measurement uncertainty would reduce the uncertainty in *CF* to 17.8% from 24.9%, which is a decrease in the uncertainty of approximately 25%. When feasible, reliable LIDAR or SODAR could be used for site assessment to significantly reduce δCF.

Acknowledgements

The authors would like to acknowledge the support of the Massachusetts Technology Collaborative for their support of the research on which this paper is based.

References

[1]Taylor, J., *An Introduction to Error Analysis*, 2nd ed., University Science Books, Sausalito, CA, 1997, Chaps. 1-4.

[2]Frandsen, S., and Christensen, C. J., "Accuracy of Estimation of Energy Production from Wind Power Plants," *Wind Engineering*, Vol. 16, No. 5, 1992, pp. 257 - 268.

[3]Quarton, D., "WIND TURBINES – Part 121: Power Performance Measurements of Grid Connected Wind Turbines," IEC 61400-12 Ed.1, 2004.

[4]Doran, J.C., and Verholek, M.G., "A Note on Vertical Extrapolation Formulas for Weibull Velocity Distribution Parameters," *Journal of Applied Meteorology*, Vol. 17, No. 3, 1977, pp. 410 - 412.

[5]Wieringa, J., "Shapes of Annual Frequency Distributions of Wind Speed Observed on High Meteorological Masts," *Boundary Layer Meteorology*, Vol. 47, 1989, pp. 85 - 110.

[6]Manwell, J.F., McGowan, J.G., and Rogers, A.L., *Wind Energy Explained*, John Wiley & Sons LTD, West Sussex, England, 2002, Chap. 2.

[7]Rogers, A.L., Rogers, J.W., and Manwell, J.F., "Comparison of the Performance of Four Measure-Correlate-Predict Algorithms," *Boundary Layer Meteorology*, Vol. 93, No. 3, 2005, pp. 243 - 264.

[8]Taylor, M., Machiewicz, P., Brower, M., and Markus, M., "Quantifying Uncertainty in Wind Resource Estimates," *WINDPOWER 2005*, AWEA, Denver, CO, 2005.

[9]Jones, S., "Examining Project Underperformance," *WINDPOWER 2006*, AWEA, Pittsburgh, PA, 2006.