

Explicit formulas for Mahler's measure

If P is a polynomial in n variables, its Mahler measure, $m(P)$ is defined to be the average of $\log |P|$ integrated over the product of n circles. This quantity appears naturally as an entropy in certain discrete dynamical systems and as a rate of growth in many other situations. When $n = 1$, there is a classical formula of Jensen that expresses $m(P)$ in terms of the zeros of P , but for $n > 1$ there is no such general formula. In the late 1970's, Smyth proved some intriguing formulas for a few polynomials of 2 and 3 variables that showed that $m(P)$ can sometimes be related to special values of Dirichlet L -functions. Recently, starting from an insight of Deninger, formulas have been proved and conjectured for infinite families of polynomials in 2, 3 and more variables relating the value of $m(P)$ to special values of L -functions of various kinds including L -functions of elliptic curves, Hecke L -functions and L -functions defined by modular forms. We will present a varied selection of some of these formulas and explain how some of them are proved.