Compositions with the Euler and Carmichael functions

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Abstract

Let $\phi(n)$ and $\lambda(n)$ be the Euler function and the Carmichal function of n, respectively. In my talk, I will study the numbers n such that $\phi\big(\lambda(n)\big) = \lambda\big(\phi(n)\big)$. Writing $\mathcal{A}$ for the set of such positive integers, I will sketch the proof of the fact that the inequalities

$$\exp\left(c\frac{\log x}{\log\log\log x}\right) < \#l(\mathcal{A} \cap [1,\ x]) < \frac{x}{(\log x)^{3/2+o(1)}}$$

hold as x tends to infinity, where c is some positive constant. Note that if p is prime such that $(p-1)/2 > 2$ is also prime, then $p \in \mathcal{A}$, and so the regular heuristics suggest that the upper bound is somewhat closer to the truth than the lower bound. The proofs use sieves, smooth numbers and distribution of primes with various arithmetic constraints.

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