Anatomy of a Number
March 13–17, 2006

Smooth numbers and delayed integral equations

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Abstract

Let P be a set of primes, and define $S(x, P)$ as the set of integers less than x having all their prime factors in P. Let $\Psi(x, P) = |S(x, P)|$, and $\pi(x, P) = |\{p \leq x : p - 1 \in S(x, P)\}$. A difficult but interesting question is to have estimates for the function $\pi(x, P)$. It's believed in number theory that the distribution of the prime factors of $p - 1$, where p ranges over the primes, should behave like the distribution of prime factors of a random integer, so that one can conjecture that $\Psi(x, P)/x \sim \pi(x, P)/\pi(x)$, under some conditions on the set P, as $x \to \infty$.

Moreover A. Granville and K. Soundararajan proved in 2002 that estimates for $\Psi(x, P)$ depend on the solutions of the delayed integral equation $u\sigma(u) = \int_0^u \sigma(u - t)\chi(t)\, dt$. In this talk we prove a general version of the above conjecture assuming the Elliot-Halberstam conjecture, study solutions of the integral equation to obtain estimates for $\pi(x, P)$, and apply our results to get asymptotics for the set of integers n less than x for which the k-th iterate of the Euler function $\phi_k(n)$ is smooth.