Anatomy of a Number
March 13–17, 2006

Lecture 1: How to count numbers wit a divisor in a given interval: main ideas
Lecture 2: Fat subset sums, clump detection and avoidance

Kevin Ford
ford@math.uiuc.edu
Department of Mathematics
University of Illinois at Urbana-Champaign
1409 W. Green St.
Urbana, IL 61801
USA

Abstract

Lecture 1:

We determine the order of magnitude of $H(x, y, z)$, the number of integers $n <= x$ having a divisor in $(y, z]$, for all x, y and z. We also study the function $H_r(x, y, z)$, the number of integers $n <= x$ having exactly r divisors in $(y, z]$, determining the order of magnitude for a wide range of x, y, z. As a consequence of these bounds, a 1960 conjecture of Erdös and several related conjectures of Tenenbaum are settled. We outline the main ideas in the proofs, in particular the appearance of "fat" subset sums and order statistics, and describe some applications.

Lecture 2:

Consider a set of k positive real numbers $x_1, \dots, x_k$ and the set S of their subset sums, i.e. S consists of all sums of subsets of the x_i, and in general S will have 2^k elements. In particular, if the x_i are the logarithms of the prime factors of a squarefree integer n, then S consists of the logarithms of the divisors of n. Now "fatten" the set S by creating a new set T which is the union of intervals, one for each point s in S, with length t and right endpoint s. The measure of T, let's call it $L(n, t)$, reflects how well the points in S are distributed. From lecture #1, we know that bounds on $H(x, y, z)$ depend on averages of $L(n, t)$ with $t = \log(z/y)$. In this lecture we describe how to obtain upper and lower bounds for this average, and where the theory of order statistics enters the picture.