

Deformation theory and local intersection numbers

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Abstract

In this talk and the sequel I wish to explain to a “general” audience some of the basic ideas and arguments underlining a purely deformation-theoretic approach to the arithmetic intersection theory calculations in the original work of Gross and Zagier. Roughly speaking, the goal is to eliminate all appeals to the “numerology” of modular curves (e.g., uniform arguments for all j -values and all CM fields) and to provide a conceptual interpretation for various formulas.

To this end, in the first talk I will first explain some basic background in deformation theory on moduli spaces, especially the Serre–Tate lifting theorem and complete local rings on modular varieties. These notions are applied to compute some intersection numbers among Heegner cycles on some modular curves in terms of isomorphism groups between pairs of elliptic curves. (This corresponds to Theorems 4.1, 5.1, and 6.4 in my paper in the MSRI “Gross–Zagier” book.)