Intersection of Arithmetic Cycles and Automorphic Forms
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L-functions, modularity, and functoriality

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Abstract

There is a very interesting, and still very mysterious, complex analytic invariant attached to an arithmetic object – its L-function. (The Riemann zeta function is an example.) There is also a family of more analytic objects that have similar complex analytic invariants – modular forms or automorphic forms. In this talk I would like to discuss both arithmetic and automorphic L-functions. I will pay particular attention to the the nature of the "Converse Theorem for $\mathrm{GL}(n)$", which morally says: any object with a nice L-function should be modular. I will explain how this leads naturally to both Langlands' conjectures on the modularity of Galois representations and Langlands' Functoriality conjecture. Finally I will discuss the Converse Theorem as a practical tool for establishing functoriality, concentrating on the cases of the lifting of automorphic forms from the classical groups to $\mathrm{GL}(n)$. The hope is that the talk will be expository, self contained, and understandable to a general mathematical audience.