Intersection of Arithmetic Cycles and Automorphic Forms
December 12–16, 2005

Borcherds products and Green functions

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Abstract

This lecture gives an introduction to the theory of Borcherds products on Shimura varieties of type $\mathrm{O}(2,n)$. Borcherds products are particular meromorphic modular forms which have a striking infinite product expansion. They are obtained from elliptic modular forms by means of a regularized theta lifting. We explain some of their geometric and arithmetic properties. We show how the lifting can be generalized to lift weak Maass forms to certain automorphic Green functions for Heegner divisors. We discuss some fundamental properties of these Green functions, for instance, their behavior at the boundary. It is shown that they define Green objects in the extended arithmetic intersection theory due to Burgos, Kramer and Kühn. Moreover, the relationship to the Green functions constructed by Kudla is briefly discussed.