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A theorem of the central limit type, in the framework of the infinite symmetric group

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It is well-known that a simplified algebraic version of either the classical or the free Central Limit Theorem can be proved via direct calculation of moments, where one manipulates sums with terms indexed by set-partitions. This line of proof doesn't in fact require a form of independence for the sequence of (non-commutative) random variables that is considered – one only needs the weaker hypotheses that this sequence of random variables is exchangeable and it obeys a certain 'singleton-factorization rule' for expectations. Under these weaker hypotheses (which are in particular implied by either classical or free independence), the determination of the resulting limit law has to be addressed on a case-by-case basis.

In this talk I present a joint work with Claus Koestler (arXiv:1807.05633) where we consider an instance of the above 'exchangeable CLT' theorem, occurring in the framework of the group algebra of the infinite symmetric group S_∞. The exchangeable sequence we consider consists of the so-called star-generators of S_∞, and the expectation functional we consider is a natural example of block-character of S_∞, parametrized by a positive integer d. Our main result is to identify precisely the limit law μ_d for this special instance of exchangeable CLT. Somewhat unexpectedly, μ_d turns out to be closely related to the average empirical distribution of a random $d \times d$ GUE matrix. In the case '$d = \infty$' of our theorem (which still makes sense, with the expectation functional considered on S_∞ being the canonical trace), the limit law μ_d becomes the semicircle law; this recovers a 1995 result of Philippe Biane.

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