

# Unitary matrix integrals and automorphisms of free groups

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Let  $w$  be a word in a finitely generated free group  $F$ .

Consider the expected value of the trace of the matrix obtained by substituting independent, Haar distributed,  $n$ -dimensional unitary matrices for the letters appearing in  $w$ . This gives a function of  $n$ . An interesting fact about this function is that it only depends on the orbit of  $w$  under automorphisms of the free group. We calculate it exactly for large  $n$  in terms of algebraic and topological properties of  $w$ .

This gives new links between the function described above, an object of study in free probability theory, and other fundamental functions on the free group, such as stable commutator length. Moreover, our calculation has allowed us to make progress on an intriguing conjecture that I'll explain in my talk.

*This is joint work with Doron Puder.*