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Introduction to quivers with relations for symmetrizable Cartan matrices

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Given a symmetrizable Cartan matrix C with left symmetrizer D and an orientation Ω one can define a Jacobian algebra $\sqcap^{(}C, D) = P(Q, W)$ over any field F. It is free as a module over a central subalgebra $F[[\varepsilon]]$, and after localizing with respect to ε we obtain a Dlab-Ringel (species) preprojective algebra of type C over the field of Laurent series $F[[\varepsilon]]$, modulo ε^k we get the generalized preprojective algebras $\sqcap(C, kD)$ defined in [GLS1].

By factoring out the arrows which go opposite to the orientation Ω we get an $F[[\varepsilon]]$ order $H^{(}C, D, \Omega)$. After localizing with respect to this becomes a species of type (C, D, Ω) over the field $F[[\varepsilon]]$. Modulo ε^k we get back the 1-Iwanaga -Gorenstein algebras $H(C, kD, \Omega)$ from [GLS1]. We will illustrate these constructions with several examples.

The Grothendieck group of the exact category of locally free $H(C, D, \Omega)$-modules has the structure of a bilinear Cartan lattice in the sense of Krause-Huberry, and all bilinear lattices can be realized in that way. The locally free rigid indecomposable $H(C, D, \Omega)$-modules are parametrized, via their class in the Grothendieck group, by the real Schur roots associated to (C, Ω) by [GLS6]. Moreover $\sqcap(C, kD)$, viewed as a $H(C, D, \Omega)$-module decomposes as the direct sum of the preprojective $H(C, D, \Omega)$-modules.

Our motivation comes from the fact that F can be any field, in particular an algebraically closed one. In that case we obtain for example the following result: The irreducible components of maximal dimension of the varieties of E-filtered $\sqcap(C, kD)$-modules have the structure of an $B_C(\infty)$-crystal.

References:

[GLS1] Christof Geiss, Bernard Leclerc, Jan Schröer: Quivers with relations for symmetrizable Cartan matrices I: Foundations Invent. Math. 209 (2017), no. 1, 61-158.

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[GLS6] Christof Geiss, Bernard Leclerc, Jan Schröer: Rigid modules and Schur roots arXiv:1812.09663v1 [math.RT], 36 pp.

This is an overview talk to joint work with B. Leclerc and J. Schröer.