Strategic fire sales and price-mediated contagion in the banking system

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Plan

Financial contagion

- Past financial crises have repeatedly shed light on the critical role played by financial institutions in propagating and amplifying an exogenous adverse shock.
- Recent example: 2007-2008 subprime crisis when a shock in a relatively small asset class, the US subprime mortgages, resulted in magnified losses for numerous financial institutions due to contagion effects.
- There are essentially two types of (potentially mutually-exciting) contagion: direct contagion and indirect contagion.

Direct contagion

- Direct contagion is the result of contractual links between financial institutions, typically debt or OTC derivatives, that generate counterparty risk.
- The failure of a given institution will trigger losses for its counterparties, potentially causing the defaults of other institutions, which will in turn trigger losses for their own counterparties and further failures etc...
- Regulators have tackled counterparty risk by introducing collateral requirements and limitations of large exposures for OTC derivatives trades.

Indirect contagion

- Indirect contagion or *price-mediated contagion* occurs through price effects, even in the absence of direct contractual links between institutions.
- A given financial institution may be forced to sell some assets, pushing prices down and generating losses for all institutions holding the same assets.
- Such forced sales are generally referred to as fire sales and typically occur at a dislocated price when a distressed institution is willing to promptly liquidate part of its portfolio.
- The price impact and destabilizing effects of fire sales may be magnified in the case where several institutions are faced with the same shock and need to liquidate assets at the same time.

Fire sales and capital requirements

- One of the main reasons why banks may engage in fire sales are regulatory capital requirements themselves.
- Under Basel accords banks are forced to maintain a regulatory risk-based capital ratio higher than a critical threshold:

$$\text{Risk Based Capital ratio} := \frac{\text{Total capital}}{\text{Risk weighted assets}} \geq 8\%$$

- A bank may need to liquidate assets in order to comply with such regulatory capital requirements after a shock.
- When the banking system is subject to a common shock, several banks may need to liquidate assets at the same time, generating feedback effects and price-mediated contagion.

Fire sales and capital requirements

- [Squam Lake Report] "because of the mark to market accounting, fire sales by some firms may force others to liquidate positions to satisfy capital requirements. These successive sales can magnify the original temporary price drop and force more sales"
- [Basel Committee] (during the subprime crise, the banking sector was forced to): "reduce its leverage in a manner that amplified downward pressures on asset prices. This deleveraging process exacerbated the feedback loop between losses, falling bank capital and shrinking credit availability."
- [European Systemic Risk Board] "Why did US sub-prime credit, which totalled 1 trillion in 2007, trigger global financial crisis - while the dot-com equity market crash, which destroyed 8 trillion of wealth in 2000, did not? ... The insight is that indirect contagion is the key ingredient through which small and local initial shocks become big, global and systemic."

New regulatory tools

To better assess the resilience of financial institutions and avoid contagion effects leading to systemic crises, regulators have

- introduced a leverage ratio, roughly speaking capital divided by total assets, to "*supplement the risk-weighted measure with a simple transparent and independent measure of risk*"
- conducted various **stress-tests** since 2011 (EBA in Europe and Fed in USA)
- designed a capital surcharge for Global Systemically Important Banks
- introduced counter-cyclical capital requirements

Aim of the present work

Our aim is to

- provide a simple theoretical framework that enables to analyze price-mediated contagion effects in the banking system following a common shock
- study the equilibrium between banks to understand the consequences of a shock and how it may be amplified due to market frictions
- show how to calibrate the model to publicly-available data and use it to draw regulatory actions

Related literature

A few related studies (see paper for detailed references):

- Cifuentes-Shin-Ferrucci (2005)
- Greenwood-Landier-Thesmar (2015)
- Braouézec and W. (2016)
- Cont and Schaanning (2016)
- literature on stress tests (see the recent survey by the BCBS 2015 "Making supervisory stress tests more macroprudential: Considering liquidity and solvency interactions and systemic risk").

Plan

Notations

- Set $B = \{1, 2, ..., p\}$ of $p \geq 2$ banks that can invest in a risky asset and in cash
- v_i : amount of cash (in dollars)
- $q_i P_t$: value (in dollars) of risky assets; q_i is the quantity (in shares) of risky assets held by the bank i, P_t is the market price of the risky asset at a given date t
- D_i : total debt (value of deposits and/or debt securities)
- $E_{i,t} = \max\{A_{i,t} - D_i; 0\} = \max\{v_i + q_i P_t - D_i; 0\}$: value of equity (or capital) at time t

Bank's balance sheet

Balance sheet of bank i at date t:

Assets	Liabilities
Cash: v_i Risky assets: $q_i P_t$	Debt: D_i Equity: $E_{i,t}$
$A_{i,t}$	$E_{i,t} + D_i$

By construction, as long as equity is positive, total liabilities must be equal to total assets so that

$$A_{i,t} = E_{i,t} + D_i \tag{1}$$

The risky asset

Assumption 1

The risky asset is a financial security issued by a non-financial institution whose price is quoted on financial markets.

- Single risky asset held by banks:
  - In practice, banks hold numerous assets. Our assumption is equivalent to assuming that banks have collinear portfolios.
  - Relevant from a regulatory stress-testing perspective. Worst case scenarios where banks' trading books are highly correlated.
- Risky asset not issued by a financial institution $\rightarrow$ the default of a bank does not impact other banks through direct contagion but only through the price of this risky asset.
- Risk-weighted asset of bank i:

$$\mathrm{RWA}_{i,t} = \alpha_i q_i P_t$$

Risk-based capital ratio

- $\theta_{i,t}$ risk-based capital ratio of bank i at date t:

$$\theta_{i,t} := \frac{E_{i,t}}{\mathrm{RWA}_{i,t}} = \frac{A_{i,t} - D_i}{\alpha_i q_i P_t} > 0 \tag{2}$$

- θ_{min} minimum capital ratio imposed by the regulator (typically 8% in Basel regulation)
- At time t, before the shock, all banks comply with the regulatory constraint:

$$\theta_{i,t} > \theta_{min} \text{ for each } i = 1, 2, ..., p \tag{3}$$

Shock on risky assets

Assumption 2

There is a shock Δ (in percentage) on the price of the risky asset at time t^+ so that $P_{t^+} = P_t(1-\Delta)$.

- Right after the shock, RBC of bank i given by:

$$\theta_{i,t^+}(\Delta) = \frac{\max\{A_{i,t^+} - D_i; 0\}}{\mathrm{RWA}_{i,t^+}} = \frac{\max\{E_{i,t} - q_i P_t \Delta; 0\}}{\alpha_i q_i P_t (1-\Delta)} \tag{4}$$

- RBC is a decreasing function of the shock size
- Bank i may thus be in one of the three following situations, depending on the size of the shock Δ

Situations after a shock

1. Solvent and complying with regulatory capital requirement, that is $\theta_{i,t^+}(\Delta) \geq \theta_{min}$
2. Solvent but not complying with regulatory capital requirement, that is $0 < \theta_{i,t^+}(\Delta) < \theta_{min}$
3. Insolvent, that is $\theta_{i,t^+}(\Delta) = 0$, i.e., $E_{i,t} - q_i P_t \Delta \leq 0$

Let

$$\Delta_i^{sale} : \quad \inf\{\Delta \in [0,1] : \theta_{i,t^+}(\Delta) = \theta_{min}\} \tag{5}$$

$$\Delta_i^{fail} : \quad \inf\{\Delta \in [0,1] : E_{i,t^+}(\Delta) = 0\} \tag{6}$$

One can show that:

$$\Delta_i^{sale} : \quad = \quad \frac{E_{i,t} - \alpha_i \theta_{min} q_i P_t}{q_i P_t (1 - \alpha_i \theta_{min})} = \frac{\Delta_i^{fail} - \alpha_i \theta_{min}}{1 - \alpha_i \theta_{min}} > 0 \tag{7}$$

$$\Delta_i^{fail} : \quad = \quad \frac{E_{i,t}}{q_i P_t} > 0 \tag{8}$$

Reaction of banks

Assumption 3

A bank which does not comply with the regulatory capital requirement can only sell assets in order to raise back its capital ratio above θ_{min}.

After a (systemic) common shock, it is easier for banks to sell assets than to issue new stocks.

Assumption 4

Static model: liquidation occurs at time $t+1$.

Let $x_i \in [0,1]$ the proportion of risky assets sold by bank i at date $t+1$, in reaction to the shock Δ at date t^+.

By convention, $x_i = 1$ for insolvent banks, ie, the bank is fully liquidated.

Price at date t+1

- Large sales of asset impact prices $\rightarrow$ use of a linear price impact model
- Φ market depth, ie, linear measure of the asset liquidity
- Denote $\mathbf{x}(\Delta, \Phi) := \mathbf{x} = (x_1, x_2, ..., x_p) \in [0,1]^p$ the vector of liquidations

Assumption 5

The price of the risky asset at time $t+1$ is equal to

$$P_{t+1}(\mathbf{x}, \Phi) = P_t (1-\Delta) \left(1 - \frac{\sum_{j \in B} x_j q_j}{\Phi}\right) \quad (10)$$

$$\frac{Q_{tot}}{\Phi} < 1 \quad (11)$$

$$\text{where} \quad Q_{tot} = \sum_{j \in B} q_j \quad (12)$$

Balance sheet at date t+1

- Balance-sheet of bank i at date t^+ right after the shock

Assets	Liabilities
Cash: v_i Risky asset: $q_i P_t(1-\Delta)$	Debt: D_i Equity: E_{i,t^+}
$A_{i,t^+} = v_i + q_i P_t(1-\Delta)$	$E_{i,t^+} + D_i$

- Balance-sheet of bank i at date $t+1$ after liquidation

Assets	Liabilities
Cash: $v_i + x_i q_i P_t (1-\Delta)\left(1 - \frac{\sum_{j\in B} x_j q_j}{\Phi}\right)$ Risky asset: $(1-x_i) q_i P_t (1-\Delta)\left(1 - \frac{\sum_{j\in B} x_j q_j}{\Phi}\right)$	Debt: D_i Equity: $E_{i,t+1}$
$A_{i,t+1} = v_i + q_i P_t (1-\Delta)\left(1 - \frac{\sum_{j\in B} x_j q_j}{\Phi}\right)$	$E_{i,t+1} + D_i$

Capital and capital ratio at date t+1

- Capital after rebalancing:

$$E_{i,t+1}(\mathbf{x},\Delta) = \max\left\{E_{i,t} - q_i P_t\left(\Delta + \frac{\sum_{j\in B} x_j q_j}{\Phi}(1-\Delta)\right);0\right\} \quad (13)$$

- Capital ratio after rebalancing:

$$\theta_{i,t+1}(\mathbf{x},\Delta) = \frac{E_{i,t+1}(\mathbf{x})}{\alpha_i q_i P_{t+1}(\mathbf{x},\Phi)(1-x_i)} \quad (14)$$

The liquidation problem

Consider the case in which there is no price impact, $\Phi = \infty$.

- When a bank is solvent after the shock, i.e., the total capital is still positive, then, there always exists a solution $x_i < 1$ such that $\theta_{i,t+1}(.) \geq \theta_{min}$.
- Each bank can choose $x_i < 1$ *independently* of the other banks, i.e., the liquidation problem is not strategic.

Consider the case in which there is a positive price impact, $\Phi < \infty$.

- The liquidation problem becomes strategic, each firm cannot decide independently of the other banks.
- Game with *strategic complementarities*. Bank i has an incentive to liquidate more risky assets when the other banks increase the quantity of risky asset they sell (monotone increasing best response). Multiple (Pareto ranked) equilibria.

Definition of an equilibrium

Let $x_{-i} \in [0,1]^{p-1}$ be a $p-1$-dimensional vector.

The vector of liquidation can be written as $\mathbf{x} = (BR_i(x_{-i}), x_{-i})$, where $BR_i(x_{-i})$ is the (unique) best response of bank i given $x_{-i} \in [0,1]^{p-1}$

Definition 1

For a given initial shock $\Delta > 0$, the vector of liquidation $\mathbf{x}^* = (x_1^*, ..., x_p^*) \in [0,1]^p$ is a Nash equilibrium if and only if for all $i = 1, 2, ..., p$:

$$BR_i(x_{-i}^*, \Delta) := x_i^* = \min\left\{x_i \in [0,1) \ : \theta_{i,t+1}(x_i, x_{-i}^*, \Delta) \geq \theta_{min}\right\} \text{ or } \quad x_i^* = 1$$

Each bank is assumed to minimize the quantity sold, given what the other banks sell.

Existence of a Nash equilibrium

Proposition 2

For all initial shock $\Delta \in (0,1)$ and market depth $\Phi > 0$, the set of Nash equilibrium denoted $\mathcal{F}_\Delta$ is not empty.

- The best response of bank i actually depends on what the rest of the banking system liquidates, ie, it is a function of $\sum_{j \neq i} x_j q_j$.
- The best response may be *discontinuous* in x_{-i}. We rely on Tarski theorem to prove the existence, which does not require any topological properties (compactness or continuity).
- When $\mathcal{F}_\Delta$ contains more than one Nash equilibrium, we shall always consider the smallest one, that minimizes the amount liquidated and that should naturally be favored by banks themselves and the regulator.

Implied shock

- At equilibrium, the (implied) shock is equal to $\Delta^* = \Delta + \left(\frac{\sum_{i \in B} x_i^* q_i}{\Phi}\right)(1 - \Delta)$.
- Δ^* represents the realized loss for the asset following the initial exogenous loss Δ and the price impact of liquidations by banks.
- When the quantities liquidated are large, ie, of the same order of magnitude than asset market depth, an initial loss may be amplified significantly.

Case of no price impact

Proposition 3

In the absence of price impact, that is $\Phi = \infty$, the proportion of risky assets liquidated by bank i is given by:

- If $\Delta \leq \Delta_i^{sale}$, then $\underline{x}_i^* = 0$
- If $\Delta_i^{sale} < \Delta < \Delta_i^{fail}$, then

$$\underline{x}_i^* = 1 - \left(\frac{1 - \Delta_i^{sale}}{1 - \Delta}\right)\left(\frac{\Delta_i^{fail} - \Delta}{\Delta_i^{fail} - \Delta_i^{sale}}\right)$$

- If $\Delta \geq \Delta_i^{fail}$ then $\underline{x}_i^* = 1$

and the volume (in \$) liquidated by bank i can be written:

$$q_i P_t \left(\frac{1 - \Delta_i^{fail}}{\Delta_i^{fail} - \Delta_i^{sale}}\right) \underbrace{\left[(\Delta - \Delta_i^{sale})_+ - (\Delta - \Delta_i^{fail})_+\right]}_{\text{Difference between two call options}} - q_i P_t (\Delta - \Delta_i^{fail})_+$$

Equilibrium for a small shock

Proposition 4

Assume that the initial shock $\Delta > 0$ is such that:

$$\overline{\Delta}^{sale} < \Delta < \frac{\underline{\Delta}^{fail} - \frac{Q_{tot}}{\Phi}}{1 - \frac{Q_{tot}}{\Phi}} \tag{15}$$

so that each bank complies with the regulatory constraint at equilibrium.

$$x_i^* = \underline{x}_i^* + \left(\frac{\underline{Q}^*}{\Phi} \frac{1 - \Delta_i^{fail}}{\theta_{min}\alpha_i(1-\Delta)} \right) + o\left(\frac{1}{\Phi}\right) \tag{16}$$

so that the total quantity sold is equal to

$$Q^* = \underline{Q}^* \times \left(1 + \frac{1}{\Phi} \sum_{i=1}^{p} \frac{q_i(1 - \Delta_i^{fail})}{\theta_{min}\alpha_i(1-\Delta)} \right) + o\left(\frac{1}{\Phi}\right) \tag{17}$$

Plan

Empirical results

- We calibrate our model for the 30 banks used in the 2015 stress test in the US (Comprehensive Capital Analysis and Review).
- We compute the Nash equilibrium for the calibrated parameters.
- We calculate the aggregate volume of liquidations in the US banking sector as a function of the initial shock Δ, for various values of liquidity parameters.
- We explore some regulatory implications of our model.

Data for US GSIBs

Bank	Total Capital	RWA	Total Assets	RBC
Citigroup Inc	165454	1292605	1842181	0,128
JPMorgan Chase &Co	206594	1619287	2572274	0,128
Bank of America Corporation	161623	1262000	2104534	0,147
HSBC North America Holdings Inc	190730	1219800	2634139	0,156
The Goldman Sachs Group, Inc	90978	570313	856240	0,160
Wells Fargo & Company	192900	1242500	1687155	0,155
The Bank of New York Mellon	21556	168028	385303	0,128
Morgan Stanley	74972	456008	801510	0,164
State Street Corporation	17914	107827	274119	0,166

Table: All quantities except the RBC are in million of dollars

Calibration

- E_t = total capital
- v = cash
- $qP_t + v$ = total assets
- αqP_t = risk-weighted assets

$$\alpha = \frac{\text{risk-weighted assets}}{\text{total assets} - \text{cash}} \tag{18}$$

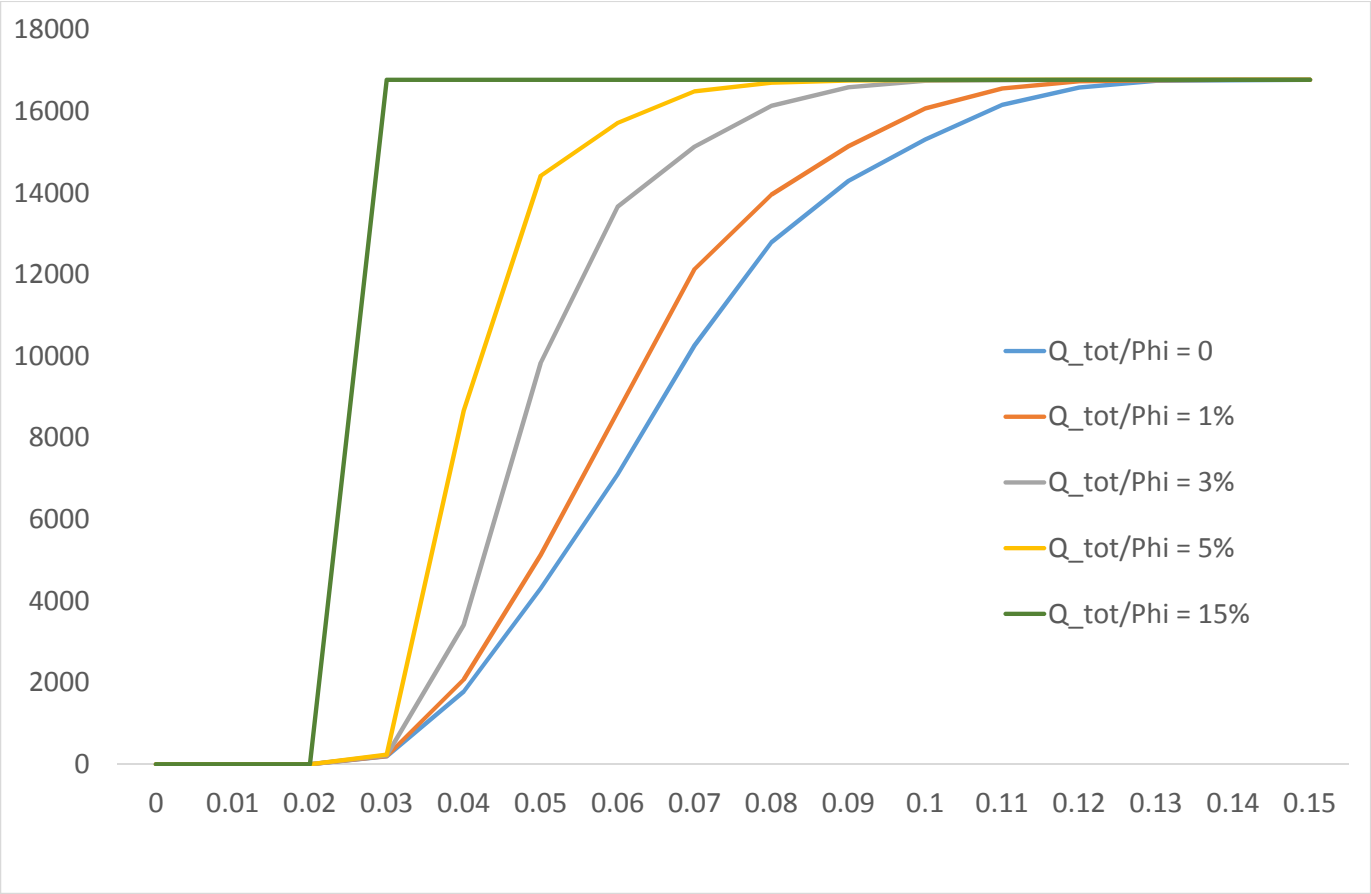
$$\Delta^{sale} = \frac{\text{total capital} - \theta_{min}\text{risk-weighted assets}}{(\text{total assets - cash})(1 - \alpha\theta_{min})}; \Delta^{fail} = \frac{\text{total capital}}{\text{total assets} - \text{cash}} \tag{19}$$

Calibrated parameters for US GSIBs

Bank	α	Δ^{sale}	Δ^{fail}
Citigroup Inc	0.7017	0.0357	0.0898
JPMorgan Chase &Co	0.6295	0.0315	0.0803
Bank of America Corporation	0.5997	0.0303	0.0768
HSBC North America Holdings Inc	0.4631	0.0367	0.0724
The Goldman Sachs Group, Inc	0.6661	0.0559	0.1063
Wells Fargo & Company	0.7364	0.0589	0.1143
The Bank of New York Mellon	0.4361	0.0218	0.0559
Morgan Stanley	0.5689	0.0503	0.0935
State Street Corporation	0.3934	0.0350	0.0654

Table: Calibrated data for US GSIBs

Fire sales in billion (Y axis). Shock size (X axis)



Best response for US GSIBs

Bank	$\frac{Q_{tot}}{\Phi} = 0$	$\frac{Q_{tot}}{\Phi} = 1\%$	$\frac{Q_{tot}}{\Phi} = 3\%$	$\frac{Q_{tot}}{\Phi} = 5\%$	$\frac{Q_{tot}}{\Phi} = 15\%$
Citigroup	0.43	0.52	0.91	1	1
JPMorgan Chase &Co	0.57	0.67	1	1	1
Bank of America	0.62	0.73	1	1	1
HSBC	0.64	0.78	1	1	1
Goldman Sachs	0.07	0.16	0.57	0.95	1
Wells Fargo	0.01	0.10	0.46	0.81	1
New York Mellon	1	1	1	1	1
Morgan Stanley	0.21	0.32	0.81	1	1
State Street	0.81	0.98	1	1	1

Table: Liquidated proportions x_i^* for $\Delta = 6\%$

Counter-cyclical capital requirements

- Consider the case of infinite market depth $\Phi = \infty$.
- A shock $\Delta = 3\%$ generates liquidations of around \$2,000Bn.
- A shock $\Delta = 6\% = 2 \times 3\%$ generates liquidations of around \$7,100Bn $> 2\times$ \$2,000Bn $\rightarrow$ liquidations are highly convex, even in the absence of price impact.
- Assume now that a regulator wants to limite liquidations to \$6,000Bn when $\Delta = 6\%$. This can be achieved by reducing (temporarily) the minimum capital requirement to $\theta_{min} = 6.75\%$.

Effect of systemic risk surcharge for GSIBs

- Consider a shock $\Delta = 6\%$ and $\frac{Q_{tot}}{\Phi} = 3\% \rightarrow$ failure of 7 banks, including 6 GSIBs.
- Assume now that GSIBs already implement in 2015 their capital surcharge (which have in fact started to be phased in in 2016): 2.5% for Citigroup and JP Morgan Chase, 2% for Bank of America ...
- In this case, only 3 banks are left insolvent after the 6% shock.
- The initial capital surcharge could be modified such that no GSIB is left insolvent.

Plan

Conclusion

- Intuitive model of a banking system which takes into account the feedback generated by banks' rebalancing.
- Enables to quantify price-mediated contagion effects.
- Can be easily calibrated to publicly-available data and used to anticipate endogenous reaction of banks following a shock in assets.
- Useful to justify measures such as counter-cyclical capital requirements or capital surcharge for GSIBs.

Thank you for your attention.