

# Integrable Lagrangians and modular forms

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## Abstract

We investigate Lagrangians of the form

$$\int f(u_x, u_y, u_t) dx dy dt$$

such that the corresponding Euler-Lagrange equations  $(f_{u_x})_x + (f_{u_y})_y + (f_{u_t})_t = 0$  are integrable. We demonstrate that the integrability conditions, which constitute an involutive over-determined system of fourth order PDEs for the Lagrangian density  $f$ , are invariant under a 20-parameter group of Lie-point symmetries whose action on the moduli space of integrable Lagrangians has an open orbit. The density of the ‘master-Lagrangian’ corresponding to this orbit is a modular form in three variables defined on a complex hyperbolic ball.