Memories of Pavel Winternitz

(J. Harnad, July 2021)

§. A half century of Pavel memories

I want to share a few reminiscences and reflections, starting from a bit before my first encounter with Pavel, and continuing through the half-century of our friendship.

It will be anecdotal, and a bit scattered, passing between personal and scientific recollections, trying to place the latter within the framework of major developments in theoretical physics of the time.

1. Acquaintance and early collaboration

1.1 Early years: high energy physics in 1968-74.

§ 1.1a. Rutherford Lab (1960's and now)
§ 1.1b. Theoretical Physics Department, 12 Parks Rd, Oxford
§ 1.1c. Prague occupation, August 1968

I first learned of Pavel in 1970 through some printed notes for a lecture series on scattering amplitude expansions he had given at the Rutherford High Energy Physics Laboratory. This was near to Oxford, where I was doing my doctorate at the Theoretical Physics Department, and my first research supervisor, **Roger Phillips**, was the head of the theory group at the Rutherford Lab. Pavel spent much of 1968-69 in the U.K., having fled his troubled homeland after the 1968 invasion and occupation of Czechoslovakia by the Soviet Union and its Warsaw Pact vassals.

When I came across his notes, he had already left to visit Lochlainn O'Raighfeartaigh at the Dublin Institute for Advanced Studies, and moved on afterwards to Pittsburgh, where he spent the next three years. So we didn't actually meet yet at that time.

§ 1.1d. Relativistic scattering amplitudes 1. 1968

I very much regretted this, since his notes were closely related to what had become the main theme of my doctoral thesis work: expansions of scattering amplitudes in terms of irreducible representations of the groups O(4) or O(3,1), with related rising families of Regge poles known then as Lorentz poles.

§ 1.1e. Lorentz Poles

I believe they had their seeds in Pavel's doctoral thesis work under Smorodinsky, and subsequent work on group theoretical analysis of scattering amplitudes. They were a helpful source, clarifying the background in a very nice way, within the broader framework of harmonic analysis on Lie groups.

At the time, the Chew-Mandelstam **analyticity-bootstrap** approach was still flourishing, and had led to the **Veneziano formula**, which provided a way of expressing high energy **hadronic scattering amplitudes** explicitly in terms of Euler's beta-function.

§ 1.1f. Veneziano amplitude

Here is a picture of **Gabriele Veneziano**, who was a young researcher at MIT and at CERN around that time.

§ 1.1g. Gabriele Veneziano (1969?)

In fact, the **Veneziano amplitude** was subsequently connected to the **dynamics of strings**, by Koba, Nielson, Nambu, Gotto, and Susskind.

§ 1.1h. History of string theory

Of course, string theory had a rebirth, years later, with a different interpretation, as a "theory of everything", including quantum gravity. But at the time, the Veneziano formula was just part of a heuristic scheme for expressing hadronic scattering amplitudes, that provided a crude skeleton of what was thought to be essential properties, but completely bypassed any quantum field theoretical foundations.

This was also the time of the break-through results on renormalizability of **non-abelian gauge theory with spontaneous symmetry breaking** demonstrated by **Gerard 't Hooft**, who was doing his doctoral thesis in Utrecht under the supervision of **Martinus Veltman**.

§ 1.1i. Unified electroweak theory
§ 1.1j. Higgs potential

They proved (with key input from **J.C. Taylor,** my second research supervisor in Oxford) that the electroweak theory based on the **Salam-Weinberg** model, with gauge group U(1) x SU(2), **spontaneously broken** to U(1) via the **Higgs mechanism** was, in fact, a renormalizable relativistic quantum field theory, which could form a viable basis for explaining the unified **electroweak interactions**. This was eventually extended to include the **strong interactions**, by adding an unbroken SU(3) **colour gauge symmetry (quantum chromodynamics),** leading to what was eventually called the **standard model**.

§ 1.k. Standard Model

With the triumphant success of **nonabelian gauge theory**, the revival of **quantum field theory** as the correct framework for high energy relativistic physics was assured, and it became clear that trying to base elementary particle dynamics solely on the "bootstrap" approach provided, at best, a picture of hadronic interactions that was too incomplete to be satisfactory.

The detailed predictions of the **electroweak** model, however, both experimental and theoretical, were spectacularly validated and relativistic quantum field theory was restored to its rightful place. The first Nobel prizes for this discovery were awarded to **Salam, Weinberg and Glashow** in 1979, while the fundamental contributions of **'t Hooft** and **Veltman** were duly recognized, though somewhat belatedly, with the award of the Nobel Prize in 1999. The **Z and W weak vector bosons** were first observed at CERN by Rubbia et al in 1982-83, leading to the award of the Nobel Prize to **Rubbia** and **Van der Meer** in 1984. With the eventual experimental observation of the Higgs particle in 2012, the last ingredient of the model was confirmed, with the award of another Nobel Prize, to **Englert** and **Higgs** in 2013.

§ 1.1l. Salam-Weinberg-Glashow Nobel Prize 1979
§ 1.1m. Rubbia-Van der Meer Nobel Prize 1984
§ 1.1n. 't Hooft-Veltman Nobel Prize 1999
§ 1.1o. Englert-Higgs Nobel Prize 2012

Back to scattering amplitudes:

The remainder of the **Standard Model**, **quantum chromodynamics**, however, which deals with the **strong interactions**, though generally accepted as valid, still leaves large gaps between the theoretical description, and what is observed in high energy experiments, such as the value of the proton mass, the short-range of the strong interactions (i.e., the "mass gap" problem) and the computation of hadronic scattering amplitudes.

- For purposes of parametrizing high energy scattering and decay amplitudes, however, the **group theoretical approach** continued to provide a useful tool for experimentalists and phenomenologists, who needed a coherent framework for analyzing and presenting their results.

§ 1.1p. Relativistic scattering amplitudes 2. 1972.

This led Pavel to a very active and productive period of work, adapting group representation expansions to such analyses, in collaboration with his colleagues **Lehar** and **Bystricky**, and later, **Macfadyen** and **Shukre**.

1.2. Our first meeting, and subsequent collaboration

My first actual meeting with Pavel was in 1975, in Ottawa, where I was spending three years as a postdoctoral fellow at the Carleton University Physics Department. He had recently taken up a post as one of ten researchers at the Centre de recherches mathématiques in Montreal.
I was then focussed on a rather ambitious project, inspired by the work of **Kibble, Utiyama** and **Sciama**, who tried to interpret **General Relativity** as a gauge theory of the Lorentz group, extending this to more general space-time symmetry groups, in particular, the **conformal group**.

§ 1.2a. Gauge theories of space-time symmetries

The gap between this somewhat esoteric project and the interests of others within my department was very great. So when I met Pavel, I felt a sense of relief in encountering a "soul-mate" in mathematical physics, who at least shared similar interests in the use of symmetries and group theory in the analysis of physical interactions.

In the following year (1976), I ended up at the CRM, rescued for continuing research through the support of Pavel, Bob Sharp and Steve Shnider, who combined their resources to provide a Research Associate position that would last for several subsequent years.

§ 1.2b. CRM: Jiri and Pavel
§ 1.2c. Steve Snider
§ 1.2d. CRM: Luc, Pavel and Jiri 1974-75

Research collaboration and friendship (1976-84)

During this period, Pavel and I began a long collaboration that was to produce some interesting results. With the passage of time, we also forged a friendship rooted in mutual understanding and respect, a shared outlook and many common interests that turned out to last a lifetime.

§ 1.2e. Symmetry reduction of tensor fields

Our first two research projects consisted of the application of some very basic differential geometric tools to two topics. The first was the parametrization of various **tensor fields** that were **invariant under space-time symmetry groups** - mainly, subgroups of the **conformal group** - with a view to using them in **symmetry reductions** of relativistic or **conformally invariant** equations appearing in mathematical physics.

This fit in with the ongoing program of classification of subgroups of the groups appearing in mathematical physics that was pursued over many years by **Pavel**, together **with Jiri Patera, Bob Sharp, Hans Zassenhaus** and others.

§ 1.2f. Pavel, Jiri, Bob Sharp and Hans Zassenhaus

The second consisted of developing a deeper understanding of the notion of **nonlinear superposition** of solutions to systems of first order ODE's, as originally introduced by **Sophus Lie**.

§ 1.2g. Nonlinear superposition

These were really no more than exercises in applied differential geometry, but both led eventually to results of more far-reaching interest.

The **symmetry reduction** approach to nonlinear PDE's arising in mathematical physics turned into an ongoing theme that Pavel was to develop, together with several other collaborators, for many years to follow. For me, it evolved into a deepened focus on its application to **gauge fields**, developed together with **Steve Shnider** (on which **Luc Vinet**'s thesis was also largely based). This eventually clarified the geometrical and topological foundations of what is now generally known as **Dimensional Reduction**.

§ 1.2.h. Dimensional reduction of gauge fields

It showed, in particular, that the **Higgs field** could be deduced as a vestigial remnant of pure gauge fields in higher dimensions, somewhat like the **Kaluza-Klein theory** derives the electromagnetic field as a vestige, under dimensional reduction, of a purely **metric field** in higher dimensions.

I would like to give a nontechnical explanation of what this involves, using a paradigm based on the **Parable of the Cave**, from the seventh book of **Plato's republic** (as translated by B. Jowett).

§ 1.2i. Dimensional reduction: Shadow dynamics: Plato's parable of the cave

Behold! human beings living in an underground cave, which has a mouth open towards the light and reaching all along the cave; here they have been from their childhood, and have their legs and necks chained so that they cannot move, and can only see before them, being prevented by the chains from

turning round their heads. Above and behind them a fire is blazing at a distance, and between the fire and the prisoners there is a raised way; and you will see, if you look, a low wall built along the way, like the screen which marionette players have in front of them, over which they show the puppets.
And do you see men passing along the wall carrying all sorts of vessels, and statues and figures of animals made of wood and stone and various materials, which appear over the wall?
And of the objects which are being carried in like manner they would only see the shadows?
To them, the truth would be literally nothing but the shadows of the images.

This may be viewed as a paradigm for dynamics following from symmetry reduction to a lower dimensional space. In the parable, the reduction is by projective invariance along the direction perpendicular to the cave wall. If, for instance, the object being carried along the path outside the cave entrance were simply a solid cube, and this were projected onto the wall, the shadow would, generally, be a hexagon. If the cube were subject to “free motion”; i.e. rotation and translation in the 3-dimensional world outside, the hexagon would appear to undergo quite complex motion, in which the side lengths and angles would change in a complicated way. But in the outer higher dimensional world, it would simply be free Euclidean motion.

This same paradigm may be applied, for example, to **nonabelian gauge theory** with **spontaneous symmetry breaking** due to the Higgs field with quartic potential. In the higher dimensional “outside world”, there would just be a **pure gauge field** present. But when reduced by symmetry; i.e. projected to the wall of the cave, the components of the gauge field along the orbits of the symmetry group would still be present in the reduced space, but would no longer be (co-)vector fields, but multicomponent scalars, and the self-interactions of these would project onto a **quartic Higgs potential**.

§ 1.2j. Higgs potential

To anticipate subsequent results, the symmetry reduction procedure, applied to free systems in higher dimensions (described by linear, constant coefficient equations) may also be used to deduce the dynamics of a variety of well-known **integrable systems**, such as the **Calogero-Moser** system and the **Toda lattice**. More generally, extended to infinite dimensions, it may be seen as the mechanism underlying the **matrix Riemann-Hilbert** “dressing method” and the **inverse spectral** approach.

The other theme: **nonlinear superposition** in systems of first order ODE’s, originally developed with **Pavel** and **Bob Anderson**, turned out to also have far-ranging consequences.

§ 1.2k. Nonlinear superposition

In its original context, it was pursued further by Pavel, together with Steve Shnider and others, who provided a general group theoretical framework for deducing such superposition principles on homogenous spaces. For me, the key feature was that there was a sort of “universal phase space”, for all nonlinear integrable hierarchies, which was a **Grassmann manifold**, and the underlying geometry consisted of lifting the natural general linear group action on a vector space, to this **Grassmann manifold.** Extended to compatible systems of PDE’s, this turned out to provide the framework underlying the theory of **Bäcklund transformations** for nonlinear integrable PDE’s within the **Zakharov-Shabat dressing method**, based on the **matrix Riemann-Hilbert** problem. (It also furnished much of the material for Yvan Saint-Aubin’s Ph.D. thesis.)

§ 1.2l. Group actions on Grassmann manifolds

When re-expressed in a linear representation, the dynamics of the corresponding operators are given by **isospectral deformations**, forming the core of the **inverse spectral method**. This geometrical framework also underlies the theory of **tau functions**, in which integrable hierarchies are viewed as generated by abelian group actions on **infinite dimensional Grassmann manifolds**.

§ 1.2m. Tau functions and their applications

I can't resist giving this little advertisement for a book on tau functions that just came out in February. It is, I believe, the only one currently available that gives a broad view of the multiple uses of tau functions, both in the modern theory of integrable systems, and in a wide range of applications.

1.3 Focus on integrable systems

Sometime around 1979-80, I remember Pavel making the remark**: "High energy theory seems too complicated and overcrowded a field to be able to make a worthwhile contribution. Why don't we switch focus to integrable systems, where there is a better chance?"** I didn't know in fact, at that time, that this was not really a new interest for Pavel, but was rooted in his earlier work with Smorodinsky, about symmetry reductions and separation of variables.

For me, this didn't really involve much of a shift, since I knew of the striking similarity between **inverse spectral methods** and **twistor-type correspondences** that had led to the construction of classical solutions of conformally invariant field equations such as the **Yang-Mills system**, its **supersymmetric extensions**, and reductions to **instanton** type solutions and **monopoles**. This was apparent already at the **classical level**, where many of the well-studied systems of integrable 2D PDE's could be derived through **symmetry reduction** from the gauge field equations. Later, it reappeared at the **quantum level**, as a deep connection between **supersymmetric Yang-Mills theory** and **integrable systems**. It remains a fundamental ingredient in understanding the ground state spectrum in **Seiberg-Witten theory** and in the **Nekrasov-Shatashvili** approach, linking the computation of N=2 SSYM correlators to conformal blocks.

Thus, following different paths, Pavel and I both ended up focussing on the theory of **integrable systems**, both **classical and quantum**, for many subsequent years.

1.3.1. Later collaboration: functional Bethe ansatz and separation of variables

§ 1.3a. Quantum integrable systems: Bethe ansatz and separation of variables

After my departure for the U.S. in 1984, our orientation diverged somewhat, and we didn't resume our collaboration until several years later, when I returned to Montreal. There turned out to be a shared interest, however, in the role of **separation of variables** in integrable systems, both **classical and quantum**. For Pavel, this was partly rooted in his long series of studies with **Bill Miller** and **Ernie Kalnins**, which drew a close connection between symmetry groups and separating coordinate systems. The approach to classical integrable systems that I pursued, on the other hand, was based on **isospectral flows in loop algebras** and **R-matrix Poisson bracket structures**. The separating canonical coordinates were defined in terms of spectral data directly on the phase space, which was not required to be a cotangent bundle.

But following the ground-breaking work of **Sklyanin**, it turned out that at least for the case of interacting quantum spin systems such as the **Sl(2) Gaudin spin chain** and **Heisenberg magnet**, the **quantum inverse scattering** approach, which led to the **algebraic Bethe ansatz**, had a **functional** formulation, which amounted to **separation of variables** in the Schrödinger picture.

Pavel and I decided to tackle this anew for this class of systems, using a more careful treatment of the Hilbert space. We found that the factorized **Bethe ansatz** wave functions could be identified with the so-called **"Niven" harmonics** on a sphere, thereby effectively proving the Bethe ansatz for this type of quantization. This work, and a generalization that allowed similar treatment of more general spin systems, led to two joint publications in 1995. These turned out to be our last collaborative works, aside from several jointly edited conference proceedings for events that we organized together. These included one volume on **superintegrable** systems, which was one of the main themes of Pavel's subsequent work, in which a link between **superintegrability** and the R-matrix approach was explained.

2. Family background, history and culture

§ 2a. Pavel Memories: beyond research

As the years passed, I got to know many more facets of Pavel's character and interests, and his wide range of knowledge. We found many points of common understanding, cultural background and outlook and grew to share a warm friendship that lasted throughout the remaining years of his life.

He was a decade my senior. His family had managed to escape some of the ravages of the second world war, and the holocaust, having lived out the war years in Britain. We both were rooted in a cultured, assimilated central European background, and were both forced, by necessity, to emigrate - in my case, as a small child, with my family, as post-war refugees, and holocaust survivors; in Pavel's case, after the brutal suppression that followed the Prague spring, also as a refugee. And we both found ourselves, in the late 1970's, at the CRM in Montreal, with a considerable overlap of shared interests in group theoretical methods in physics.

§ 2b. Pavel background: Moriz Winternitz

Pavel came from a notable academic family, that had a record of high achievement in his recent forebears. These included, amongst others, Moriz Winternitz, a renowned Indologist who was for many years Professor at the Karl-Ferdinands Universität in Prague and who, together with all the Winternitz family, was on cordial terms with Einstein during the 1911-12 period of his professorship at the university. There were also distinguished medical practitioners and researchers in the family, including Pavel's father. This undoubtedly had a strong impact on his outlook and expectations, providing an added determination to achieve a degree of distinction in his own work worthy of his family background. This he did, throughout his many years of research work, which was recognized by the award of many academic prizes and distinctions.

§ 2d. CAP-CRM Prize
§ 2e. Wigner Medal 1
§ 2f. Wigner Medal 2

3. Shared experiences

§ 3a. Canoe pic

Over our many years of friendship, we shared in a variety of adventures that extended well beyond the scientific. In the early years, Pavel and his sons, together with Bob Sharp, used to organize wonderful **canoe expeditions** which, together with the more experienced adventurers, like Pavel, his boys, Bob and his son, Yuri Berest and other friends, also welcomed some totally unskilled, inexperienced, and clumsy novices, such as myself, Oleg Strassburger and others. The trips were usually at Le Domaine in La Verendrye Park, which consists of a complex network of rivers and lakes, including many stretches of rapids, where we enjoyed the peace and serenity of nature, camping at various beautiful isolated campsites, on islands, or by pristine lagoons and beaches, waking to the haunting sound of loons or the magnificent sight, at sunrise, of moose standing majestically, by the shore of our campsite, immersed in the clear waters of the lakes.

In the evenings we had great campfire meals, and shared in storytelling and discussions, about experiences, memories and ideas. In the daytime, with all the baggage that we carried, and two people per canoe, we had some heavy paddling to do, and sometimes had to portage goods around rapids. Usually, we would return and run the rapids with just the canoes and pairs of canoers – which often was quite a hair-raising thrill, at least to the less skilled amongst us. Once, together with my daughter, who was one of the more sporting types amongst us, we managed to flip a canoe in a particularly rapid stretch of water, losing it completely to the current, while we just helplessly floated down the rapid surface of the water.

On another trip a further spill occurred that was a bit more embarrassing, in a relatively easy stretch, which only posed the challenge of a ninety degree turn, in relatively rapid current, while passing under a bridge. But a pair of somewhat overweight canoers (myself and Oleg) managed to lose control, and ended up, with all our belongings, in the water. This was silly enough, but to our great shame, when we had recovered everything and were again serenely canoeing down a quiet stretch, with our wet things stretched over the bow to dry in the sun, we managed somehow to again tip the boat over, this time losing some small clothing items for good. But none of these minor mishaps diminished in the least the good spirits we enjoyed; it just added further colour to the adventure. Pavel and the boys took it in good spirit, forgiving our ineptness, and we all laughed it off while drying our clothes by the campfire.

There were further adventures shared, in a variety of more or less exotic places, in particular, at the various venues where we had the good fortune to be for annual conferences: like Crete (Kolymbari), Sardinia (Cala Ganone), Sabaudia, Salento (Galipolli), and elsewhere. The pictures in the slide show to follow give a glimpse of this, as well as some other celebrations and adventures.

4. A tribute to Pavel

§ 4a. A half-century of Pavel memories

Beyond his scientific contributions, Pavel's kindness, humanity and generosity were greatly valued and appreciated by all who knew him, and there were very many who benefited from his encouragement and support. The range of his interests and skills was very wide, encompassing a cultural, scientific and human scope of unique breadth and depth. Pavel will be fondly remembered by us all, and sorely missed.

§ 4b. Slide show 1965-2021