Memories of John McKay.

John McKay was a good friend, a distinguished colleague, and a very special person, with a unique character.

He was remarkable in many ways, both as an imaginative, creative mathematician, well-known for his incredible knack of discovering unexpected relations between seemingly unrelated fields, and as a clear-minded, perceptive thinker, with great honesty and integrity. It followed from this that he could not let things that he perceived as wrong to just go by, without probing their causes. He was also very knowledgeable on a wide range of social, historical, scientific and cultural matters, and it was always interesting and stimulating to communicate with him.

I would like to recall some of the amazing things he discovered, which earned him world renown as a creative mathematical thinker, quite different in his approach from others. His contributions within his fields of specialty, which included, in particular: coding theory, group theory and number theory, were so unique, they were viewed with a certain awe by those who knew of them, even many who could not grasp their detailed significance. He was greatly respected and appreciated throughout the worldwide mathematical community.

There were two special domains, both still under current active study, which came into existence because of his unique insights. One is known as "Monstrous Moonshine" (following the coinage of John Conway and Simon Norton); the other, as the "McKay correspondence". Both relate to properties of transformation groups and symmetries, and both were inspired by John's remarkable observation of completely unexpected relations between quite distinct domains which, if not for his "second-sight", may never have been recognized. They both bridged what were, a priori, completely unrelated topics concerning invariants and geometry, and led to a huge variety of results and techniques that clarified and rendered more understandable their deeper meaning. And yet, they both remain something of a mystery, and continue to be a source for further development and generalization.

"Monstrous Moonshine" relates linear representations of the biggest *sporadic* finite simple group, the "Monster", to *modular functions*, which have a deep number theoretic meaning. This was explained through the work of Richard Borcherds (who was awarded the Fields medal for it in 1998) and others, which implemented the methods of *vertex algebras*, originating in constructions from quantum field theory, string theory and integrable systems. The "McKay correspondence" arose from another of John's remarkable observations, relating the venerable topic of *Platonic solids*, and their symmetries, to yet another, completely different group theoretical and combinatorial structure: *Dynkin diagrams*, which characterize certain continuous transformation groups and reflection groups. In each of these developments, John was responsible for pointing out the "tip of an iceberg": something quite unusual, that subsequently turned out to have deep and remarkable structure beneath.

The importance of these discoveries was accorded due recognition. In 2000, he was elected Fellow of the Royal Society of Canada, and subsequently was awarded a number of prizes and distinctions (in particular, Canada's highest award for outstanding contributions to mathematics, the CRM-Fields-PIMS prize, in 2002-3) (http://ctr.concordia.ca/2002-03/Jan 16/06-mckay/index.shtml)

Numerous international conferences were dedicated to his work, such as the 2007 CRM conference: "Groups and symmetries: from the Neolithic Scots to John McKay", sponsored jointly by Concordia and the CRM,

http://www.crm.umontreal.ca/McKay07/index_e.shtml and others

https://www.math.nagoya-u.ac.jp/~y-ito/McKay2012.html https://indico.ipmu.jp/event/314/overview"

His ideas continue to be an inspiration for ongoing research in these fields.

Although our backgrounds and research orientation were rather different (my own being rooted in mathematical physics), we shared a common fascination with symmetry groups, and their appearance in a variety of settings. This led to a brief but fruitful collaboration, on how certain modular functions that play a central role in "Monstrous Moonshine" also provide solutions to a class of differential equations that are of interest in theoretical physics (gauge field theory and the theory of integrable systems) and in geometry (orthogonal coordinate nets and Frobenius manifolds). John was always looking for some new viewpoint (preferably, rooted in physics or geometry) from which to better understand the things that intrigued him. In particular, I think he was never quite satisfied with the explanation for Monstrous Moonshine based on vertex algebras, and always hoped to find some natural geometric or physical basis for the connection, such as a differentiable manifold (possibly, 24 dimensional) upon which the "Monster" acts as a group of symmetries. Maybe, someday, this will be found.

Besides his remarkable mathematical insights and contributions, it would be very incomplete to not say a bit more about John McKay's human qualities, his very high ethical integrity, and his wonderfully wide scope of interests and knowledge, on historical, scientific and cultural matters. In particular, he had a very deep sense of the rights and wrongs regarding how things are done in an academic institutional setting, and never ceased to question things that seemed to him as unjustified or inappropriately prioritized. He was also very frank in expressing such concerns directly to some academic administrators of the time. Though this was always done with courtesy, and in a constructive spirit, his persistent enquiries spurred a rather negative backlash, with harsh measures following. There was a regrettable period in which he was quite unfairly deprived of some of the most elementary benefits due to all faculty members. Fortunately, before he retired, some constructive changes in administration took place which led to a period of improvement in faculty relations. David Graham became Dean of the FAS at Concordia, from 2005 to 2008, before becoming Provost, and did much to countervail the harsh treatment to which John had long been subjected, making sure that he was duly recognized and celebrated by the University, as he had long deserved, for his many distinguished contributions and years of service.

John was a remarkable, many-faceted person, with considerable courage, and a wonderful spirit of endurance. In the final period of his life, this helped carry him through a series of health crises which, during the last four years, sadly, confined him to a permanent care institution. We used to visit him regularly, until the pandemic crisis put a stop to that. After that, we were limited to weekly Zoom conversations, or just telephone contacts. But throughout this time, despite the recurring crises, his company was always enjoyable, and he was able, till the very last year, to keep in touch with other colleagues and friends, with the sustaining support of his devoted wife Trinh. He remained aware of what was going on in the world, and interested in it, till the end, and was still buoyed up by the hope or glimmer of some new mathematical discovery.

He was a good friend and a source of inspiration to the many who knew him, and admired him. He will be sorely missed, and fondly remembered.

John Harnad Montreal, May 2022