

## *In memoriam*

# S. Twareque Ali (1942 – 2016)



S. Twareque Ali

*A Book of Verses underneath the Bough,  
A Jug of Wine, a Loaf of Bread – and Thou  
Beside me singing in the Wilderness –  
Oh, Wilderness were Paradise enow!*  
(The Rubáyát of Omar Khayyám, Quatrain XII,  
English transl. E. FitzGerald, 5th ed., 1889)

It is with great sorrow that we communicate the passing of Syed Twareque Ali, a long-time member of the CRM, Professor at Concordia University, eminent researcher in mathematical physics, good friend and valued colleague. The following is a collection of personal recollections by some of his closest friends and collaborators, who had the privilege of sharing thoughts and experiences with him over many years, and an overview of his career and his scientific contributions.

## Personal recollections

### Jean-Pierre Antoine (Louvain-la-Neuve)

I started to collaborate with Twareque in 1989, on the topic of coherent states. He was originally referred to me because of difficulties encountered in the decomposition of a phase space representation of the Poincaré group. The solution to this was quickly found, just replacing a bounded Hilbert space operator by an unbounded one, and this began a wonderful collaboration that continued till the sadly unexpected, premature end of his life. In the following year, Jean-Pierre Gazeau joined us and thus started our triangular collaboration that led to eleven joint papers and two joint books.

Twareque travelled a lot; we kept meeting each other all over the world, until the most recent times. We often attended the same meetings, in addition to visiting each other at our home universities, including being respective invited professors. To name a few of the places we travelled together, there were Québec, Germany (Clausthal), Poland (Białowieża), Cuba, France, Italy, Bangladesh (Dhaka), China (Tianjin), India (Bangalore), and many others.

Working with Twareque was always a rewarding experience. Even if his papers were sometimes dense and compact, his clean, polished mathematical style was always a pleasure to read, and all his papers have a luminous clarity. His nonscientific writings have the same quality, including his poetry; I remember a memorable poem composed by him in Białowieża! His talks were always extremely clear and pedagogically well thought out, and his ideas were often thought-provoking.

Altogether I feel fortunate to have been able to work with him for so long, and so surely do all his other collaborators. Everyone will remember his characteristic whole-hearted laughter, inviting everyone to share his great and subtle sense of humour, and enjoyment of the moment, and of life.

### Jean-Pierre Gazeau (Paris-Diderot)

I met Twareque through Jean-Pierre Antoine at the end of the 1980s, and we started a long and fruitful collaboration, which continued to the last. It was always extremely pleasant to work with him because of his intelligence and extreme tolerance for the ideas advanced by others. Open mindedness was one of his great qualities. We still had new projects planned, and I had planned to come to Montréal this May to develop one of these with his recent Master's student Das. I believe it was my mention of Twareque's name to Anatol Odziejewicz in '91 that led to his subsequent strong and constant involvement in the organization of the Białowieża series, with so many excellent remembrances.

### Gerald Goldin (Rutgers)

Twareque Ali and I met in Clausthal, Germany during the 1980s, invited by Heinz-Dietrich Doebner to the Arnold Sommerfeld Institute. Each summer from 1992 to 2015, nearly without exception, we would see each other at the Workshop on Geometric Methods in Physics in Białowieża, Poland. With our spouses we toured Beijing and visited the Great Wall of China after the Group Theory Colloquium in Tianjin in 2012 (where he was honoured on the occasion of his 70th birthday). We shared family weddings and other events. Twareque was not only a colleague and collaborator, but a close personal friend, an intimate confidant. He was someone with whom one could discuss the meaning of life's joys and disappointments without self-consciousness.

With his serious expression, his black hair and beard only lately streaked with grey, he looked thoughtful and wise — but his eyes twinkled, and he loved to laugh. He knew how to live life, to find humour in its seriousness. He would quote poetry extensively from memory, in Bengali, Italian, German,

and of course English. He loved Omar Khayyam's Rubá'yát, in the Edward FitzGerald translations. And his laughter always restored balance. He was especially fond of the novel *Small World* by David Lodge, which satirizes the sometimes pretentious academic scene we both knew so well. We imagined we could recognize Lodge's characters in people we knew, including (of course) ourselves.

And Twareque was a deep thinker. He believed deeply in peace, and gave generously of himself to the less-privileged in the world. For him, the truths of science were part of the ever-unattainable beauty for which he yearned, for which all of us yearn. He sought scientific truths through mathematics, especially an understanding of the mysteries of quantum mechanics. Influenced deeply by his teacher Gérard Emch, he in turn inspired numerous students and colleagues. Though Twareque has gone now, his inspiration, gentleness, and humanity will live on for many generations.

### **John Harnad (Concordia)**

Twareque was a cherished friend, a unique and wonderful human being, a highly esteemed researcher and valued colleague. We shared many things over nearly four decades of friendship. All who knew Twareque could appreciate his abundance of wisdom, humour, generosity, kindness, tolerance, and enduring, simple goodness of nature.

Science and friendship were both very important to him. He had a large network of colleagues and friends around the globe with whom he kept in touch in his many travels, and shared his enjoyment of life, and his fascination with the variety and richness of all he encountered. He saw something positive everywhere, and had a genuine love of different cultures, personalities and environments, finding something of value to enjoy and celebrate everywhere he went and in all things he did.

It was characteristic of Twareque to be constantly on the move, travelling around the world, visiting friends and family, attending scientific conferences, sharing his thoughts and experiences generously with others, while always further augmenting his vast store of knowledge, his awareness of things of beauty, and the poetry and richness of life.

Sharing his laughter, good humour and enjoyment of the diverse environments he relished and the friendships he formed helped lift everyone's spirits. The wide scope of his knowledge, not only of science, but the broad range of human experience, things of learning and of beauty, poetry, art, diverse cultures and languages, together with the benefits of humour, combined to give him a unique perspective on life. We all benefitted from his positive spirit, his learning, and his appreciation of the variety and richness of life.

In addition to his scientific work, he worked relentlessly, discretely, unobtrusively, in helping others in need; this was simply second nature to him, an essential ingredient in his life. Whether those who came to Canada to continue their lives, or remained in his home country of Bangladesh, or in Cuba, or elsewhere, where circumstances made their lives much more difficult than here, he was always helping those facing the

challenges of life. He made it his ongoing mission to assist students, colleagues and others in overcoming the obstacles they faced and making possible the fullest use of their abilities. Throughout the many crises that occurred in his country of origin, he remained a lifelong, devoted, generous source of help to those who needed it.

I had the good fortune of sharing and enjoying his friendship, his company and good nature for many years, occasionally on travels, at various events and places around the world, as well as at home. It was both uplifting and cheering to be with him. I greatly benefitted from the generous gift of his friendship, his wisdom, his kindness and his enjoyment and celebration of life. Twareque was a unique, kind and good human being, an irreplaceable friend and highly valued colleague. He will be sorely missed.

### **Mourad Ismail (Central Florida)**

I first met Twareque Ali when we both arrived in Toronto in the fall of 1975 to start a one-year postdoctoral fellowship. We shared an office and taught two different sections of a calculus class concurrently. Twareque had such a nice personality that we became friends very quickly and this led eventually to a lifelong friendship.

In those early days, it seemed that our research interests were far apart, but later Twareque and I met at various conferences and exchanged ideas about our work. Fast forwarding thirty-five years, when I visited Montréal in the summer of 2011, Twareque told me about coherent states and proposed a very interesting question involving orthogonal polynomials. We then began a very enjoyable and fruitful working collaboration on this problem. He also invited me to give a talk at a special session he was organizing at a meeting on Group Theoretical methods in Physics in Tianjin, China. At first, I was hesitant, but Twareque insisted that it would be worthwhile to attend. In the end, it turned out to be one of the most productive meetings I ever attended. Thanks to Twareque, working on this topic led to a goldmine of new research results relating our overlapping interests in orthogonal polynomials and coherent states, including two joint papers on the subject.

Twareque will live in my memory for the rest of my life, as a friend, collaborator, and a very caring and wonderful human being.

### **Anna Krasowska (Vanier College) and Renata Deptula (John Abbott College)**

Twareque Ali was our Ph.D. supervisor at Concordia University; we came to know him as "Dr. Ali." He will forever remain in our hearts as Dr. Ali, and this is the way we remember him, a fatherly figure. He had a far deeper impact in our lives than the role of supervisor.

He was always a very patient, encouraging teacher and so generous with his time. He understood so well all the needs and challenges of students in a foreign land, undoubtedly because this had been his experience more than once in his own life. Our meetings, while mostly filled with discussions about

wavelets or Wigner functions, always had time reserved to talk about our new experiences in Montréal. If anything in our lives became too complicated it was a clear sign we needed to talk to Dr. Ali. Every meeting with him provided a big dose of encouragement and new energy, never accompanied with any criticism or judgment.

All his students knew how fond he was of visiting new places, meeting new people, establishing new friendships. And as it was with everything else, whatever he cherished, he shared: whether we travelled to Cuba, Mexico or Germany we felt part of a big family, because we were students of Dr. Ali.

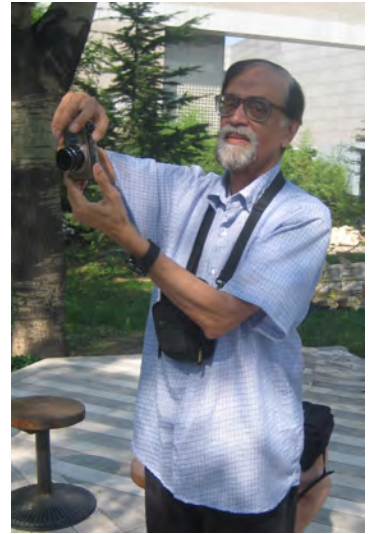
It has been a great privilege to have known Dr. Ali. He will always be an inspiration to us as a wonderful teacher and a virtuous person.

### Reinaldo Rodriguez Ramos (Havana)

Twareque Ali's contributions to the scientific development of the Faculty of Mathematics and Computer Science at the University of Havana went well beyond the co-organization of conference series. Since 1998, he began collaborating with the faculty, encouraging the participation of teachers in research and in the development of the research on wavelets and

its various applications. He provided further ongoing help to the University and the Faculty of Mathematics and Computer Science by organizing donations of computer equipment and bringing international experts to deliver specialized courses and share their knowledge and experience with the Cuban community of mathematicians. In 2009, the University of Havana honoured him with the special award of Invited Professor of the University of Havana.

He was a man of great human sensitivity, and a deep thinker who could often come up with the best solution to any problem. He was also a cherished special friend of my family. We regret the loss of a great friend and a great man of science.



## Overview of Twareque Ali's scientific contributions and career

*Nitya kaaler utshab taba*

*Bishyer-i-dipaalika*

*Aami shudhu tar-i-mateer pradeep*

*Jaalao tahaar shikhaa* (Tagore)

(English transl.: "Thine is an eternal celebration. . . A cosmic Festival of Lights! . . . Therein I am a mere flicker of a wicker lamp. . . O kindle its flame, (my Master!)")

Twareque Ali's life and career included a range of contrasting experiences. Born a citizen of the British Empire, he acquired a succession of three further nationalities: Pakistani in 1947, Bangladeshi in 1971, and finally Canadian.

After completing his M.Sc. in Dhaka, Bangladesh in 1966, and a Ph.D. in 1973 in Rochester, USA under the supervision of G. Emch, he occupied positions successively at ICTP (Trieste, Italy), Univ. of Toronto, Univ. of Prince Edward Island, and TU Clausthal (Germany), before settling down, in 1981, in Montréal, first as Assistant Professor (1981–1983) in the Department of Mathematics and Statistics, Concordia University, then Associate Professor (1983–1990) and finally Full Professor (as of 1990).

Twareque Ali was a world-recognized leader in at least three domains of current research in mathematical physics: quantization methods, coherent states and wavelet analysis; he managed to build important bridges between these, as well as publishing a large number of ground breaking research articles and widely appreciated monographs on these subjects.

He was a member of the Mathematical Physics Laboratory of the Centre de recherches mathématiques since its inception in 2001, and contributed actively to its range of scientific activities. He was also Concordia's representative to the ISM for many years, before becoming ISM Director in 2003. Over the years, he was an invited professor at many foreign universities throughout the world and was in constant demand as an invited speaker at international conferences.

His main scientific interests can be aptly summarized by the key words that represent his favourite topics: phase space, quantization, positive operator valued measures, reproducing kernels, coherent states. He began a close and fruitful collaboration in the late 1980s with two of his principal co-authors — Jean-Pierre Antoine and Jean-Pierre Gazeau — that continued throughout his lifetime. As described in the personal recollections above, it is on the last of these topics that they collaborated for the first time and this continued up to the present day. Along the way, eleven papers were jointly written (plus four with JPA alone and ten with JPG), three books edited (the Białowieża conference proceedings), two research monographs [2, 3] were co-authored, and a recent special volume co-edited, together with F. Bagarello [1]. Throughout these years, Twareque relentlessly sought to instill mathematical rigour in the field of coherent states, something he felt was needed, but too often missing in some of the published literature on quantum optics.

More specifically, in the 1970s and 1980s, he devoted much of his attention to measurement problems in (fuzzy) phase space and stochastic, Galilean and Einsteinian; quantum mechanics. This was partly in joint work with E. Prugovecki in Toronto (which led to a memorable and lasting dispute with Gerald Kaiser), G. Emch in Rochester and H.-D. Doebner in Clausthal. His 1984 *Stochastic Localization, Quantum Mechanics on Phase Space and Quantum Space-Time* was a pedagogical landmark for the domain. He then gradually focused on coherent states for the Galilei and the Poincaré groups and other semi-direct product groups. One of his first works in this direction was with S. De Bièvre. A notable result of [2] was the extension of square integrability of group representations to homogeneous spaces and the introduction of continuous frames in Hilbert spaces, the key to many applications, including wavelets.

One of his best known joint papers with Antoine and Gazeau was the 1993 *Annals of Physics* *Continuous frames in Hilbert space*. Twareque subsequently began to be interested in quantization methods, mostly Berezin or coherent states quantization, and in the mathematics of signal processing. In the last few years, his focus of interest included noncommutative quantum mechanics, quaternionic Hilbert spaces and complex orthogonal polynomials, on which a further productive collaboration began with his old friend from Toronto days, Mourad Ismail. (See M. I.'s personal recollections above.)

Travelling widely and frequently in his professional life, he always managed to combine these travels with a keen appreciation for the cultural and human milieu in which he found himself.

Another important part of Twareque's working life was the organization of scientific meetings on topics generally related to his research interests. Two series of conferences are notable. First, the Białowieża Workshops on Geometric Methods in Physics, spearheaded by their charismatic founder and chief organizer, Anatol Odziejewicz. These take place annually in a region of Poland bordering on Belarus that is famous for its local culture and natural beauty, the wild forests, bison, and local artisanal crafts. The workshops are characterized also by the abundance of good food (as well as vodka), good spirits and active participation of several Russian mathematicians of the highest caliber. From the XIth meeting, in 1992, onward, Twareque was instrumental in transforming what was earlier a small local workshop into a full-fledged international event, which is still going strong. (The last edition was Nr. XXXIV, in June, 2015.)

A further remarkable achievement was the series of workshops in Havana, Cuba, organized jointly by Concordia University and the Universidad de La Habana, in which the leading spirit was Twareque's longtime friend Reinaldo Rodríguez Ramos. As member of the Organizing Committee and founder of the

Workshops I–XII held in the Faculty of Mathematics and Computer Science at La Habana from 1998 to date, Twareque was one of the driving forces of the meetings. He succeeded in attracting a number of distinguished participants of international standing to those beautiful surroundings and almost single-handedly arranged for the publication of the proceedings. Through the Department of Mathematics and Statistics and the Center for International Academic Cooperation at Concordia University, he contributed valuably to the development of international cooperation and scientific collaboration with the Universidad de La Habana.

More recently Twareque also became a faithful participant and contributor at the school and workshops on mathematical physics (COPROMAPH) organized in Cotonou, Bénin, by Norbert Hounkonnou.

Throughout his career Twareque was deeply devoted to the teaching and supervision of students, both undergraduate and graduate, and was greatly appreciated by all his students for his patience, insight and generous help. Chairman of the graduate studies committee at Concordia's Department of Mathematics and Statistics for many years, nine Doctoral degrees were completed under his supervision or co-supervision and an equal number of Master's degrees. He was also, from 2003 till 2006, Director of the Institut des sciences mathématiques (ISM), the inter-university consortium responsible for coordinating graduate studies at Québec universities. During his term as Director, he contributed much to the enhancement of both the quality of graduate education in mathematics at Québec universities and the level of its support. He helped enhance the ISM outreach to schools and CEGEPs, as well as to improve and advance the rationalization of pedagogical resources, the level of cooperation between different university groups, and the recruitment of high quality students from overseas.

All his scientific colleagues and collaborators would agree that Twareque had a very deep and subtle understanding of Quantum Physics, both in its foundations and its working mathematical tools. It was extremely pleasant to interact and work with him, not just because of his intelligence, original insights and innovative contributions, but also his modesty, consideration and generous interest in the ideas advanced by others. Open-mindedness was one of his greatest qualities.

- [1] S.T. Ali, J.-P. Antoine, F. Bagarello, and J.-P. Gazeau, eds. *J. Phys. A: Math. Theor.* 45.24 (2012): *Special Issue on Coherent States: Mathematical and Physical Aspects*.
- [2] S.T. Ali, J.-P. Antoine, and J.-P. Gazeau. *Coherent States, Wavelets and Their Generalizations*. 2nd ed. Theoretical and Mathematical Physics, volume. New York: Springer, 2014.
- [3] J.-P. Antoine, R. Murenzi, P. Vandergheynst, and S.T. Ali. *Two-Dimensional Wavelets and their Relatives*. Cambridge: Cambridge Univ. Press, 2004.