This document contains a suggested bibliography and references for the topics that we cover on the minicourse.

General recommendations for standard ergodic theory books can be found on the web in math.stackechange and mathoverflow. For infinite ergodic theory see Aaronson's classical book [1].

Examples.

- For a general introduction of translation surfaces and interval exchange transformations go to Yoccoz's notes (Pisa Lectures). This notes provide a good formal definition of the "unfolding trick". Concerning the conjecture on periodic orbits in polygons, we recommend R.E. Schwartz's paper [21] and references within.

  For a general introduction to Veech's work, groups and "very symmetric" translation surfaces go to Hubert and Schmidt's *An introduction to Veech surfaces* [12].

- The details of the proof that every irrational polygon generates via the unfolding trick a Loch Ness Monster can be found in [24]. An unexpected connection between polygonal billiards and holomorphic foliations is detailed in [23].

- For details on periodic wind-tree models we recommend [11] and Delecroix-Zorich's Cries and whispers in wind-tree models.

- The infinite staircase was introduced and studied in [9]. The infinite staircase is part of a more general class of infinite-type translation surfaces called, $\mathbb{Z}$-covers, which were introduced in [10].

- Baker's surface was introduced by R. Chamanara in [4]. The relation between the baker map and infinite-translation surfaces is explained by him, Gardiner and Lakic in [5].

- Thurston-Veech's construction for infinite bipartite graphs was defined by Hooper in [8]. This will be a recurrent reference, for the paper is vast.

Basic definitions.

- There are some other nice introductions to *compact* translation surfaces that stress the importance of both the geometric and analytic point of view of these objects. These include [6], [26], and [27]. Here one can find a detailed discussion about finite cone angle singularities, the flat metric, the translation flow, *etc.*

- Wild singularities of translation surfaces and their affine invariants were introduced in [3]. This kind of singularities and their invariants were studied in detail in Anja Randecker's Phd. Thesis.

- Our main recommendation to understand Veech's dychotomy and Veech groups of compact translation surfaces is Hubert and Schmidt's survey [12]. However, you can also consult M. Moeller's *Affine groups of flat surfaces* [17].
- The classification theorem for infinite type surfaces can be found in [20].

Veech groups.

- Veech groups of translation surfaces arising from irrational billiards were computed in [25]. The proof that every countable subgroup of $\mathrm{GL}_+(2,\mathbb{R})$ can be realized from as the Veech group of a translations surface structure on the Loch Ness monster can be deduced from the main theorems of [18]. This result was extended to an infinite family of infinite genus translation surfaces in [19]. We refer to this last paper also for a general discussion on the problem of realizing any countable subgroup of $\mathrm{GL}_+(2,\mathbb{R})$ as the Veech group of a translation surface on an infinite-type translation surface of some fixed topological type. Veech groups of Thurson-Veech surfaces arising from infinite graphs are computed in [8].

  An interested reader can consult Hooper and Weiss's results on Veech groups on $\mathbb{Z}-covers$ [10]. These results have been generalized by J. Cabrol on his Phd. Thesis.

- The relation between the ergodicity of the translation flow and the Veech group for surfaces of finite area (but infinite topological type) is done by R. Trevino in [22].

Recurrence and ergodicity.

- A detailed discussion on how primitive classes in $H_1(M,\Sigma;\mathbb{Z})$ define $\mathbb{Z}$-covers of M can be found in the first pages of [10]. In this paper one can also find the proof of the recurrence of the translation flow on $\mathbb{Z}$-covers, which depends on a classical result by Kerckhoff-Masur-Smillie, see [14].
- The recurrence of skew products over ergodic transformations was independently obtained by Atkinson [2] and Krygin [20]. The geometrical criterion developped by Avila and Hubert to obtain the recurrence of wind-tree models can be found in this preprint.
- The ergodicity of the translation flow for the infinite staircase follows from the work of Conze on cylinder flows over irrational rotations [15]. The most general result concerning the ergodicity of the translation flow of $\mathbb{Z}$-covers is contained in the work of Hubert and Weiss [13]. Their results are built on Masur's criterion and Schmidt's criterion for ergodicity of skew products using essential values (see references within the aforementioned paper).
- The non-ergodicity of the translation flow for wind-tree models can be found in [7]. Examples of non-periodic wind-tree models whose translation flow is ergodic can be found in [16].
- The classification of all locally finite ergodic invariant measures for the translation flow F_θ^t for countably many directions θ can be found in the work of Hooper [8].

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