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On the number of nodal domains for flat tori

Résumé/Abstract: We are interested in the eigenvalues and eigenfunctions of the Laplacian on the flat tori obtained when quotienting the plane by a rectangular lattice. The eigenvalues, and an orthogonal basis of eigenfunctions, are of course explicit but complex nodal patterns can occur because of high multiplicities when the ratio of the sides in the lattice is rational.

We will consider some results concerning two questions. The first was initiated by Pleijel in 1956, and consists in finding eigenfunctions associated with the k -th eigenvalue and having k nodal domains, which is the maximal number of nodal domains allowed by Courant's theorem. In the case of a square torus, we will see that this occurs only for the first and second eigenvalues.

We will also consider some results concerning a question raised by T. Hoffmann-Ostenhof in 2012: are there eigenfunctions having an odd number of nodal domains. We will see that the answer depends on the ratio of the sides in the rectangular lattice. The answer is in particular negative in the case of a square torus.