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Integral equation methods for spectral problems

Résumé/Abstract: In this talk we present a range of numerical methods which, based on use of Green functions and integral equations, can be applied to produce solution of Laplace eigenvalue problems with arbitrary boundary conditions (including, e.g., Dirichlet/Neumann mixed boundary conditions) and in arbitrary domains (including e.g. domains with corners and multiply connected domains).

As part of our presentation we present newly obtained characterizations of the singularities of solutions and eigenfunctions which arise at transition points where Dirichlet and Neumann boundary conditions meet; the numerical methods mentioned above rely on use of these characterizations in conjunction with the novel Fourier Continuation technique to produce solutions with a high order of accuracy. In particular, the resulting method exhibits spectral convergence for smooth domains (in spite of the solution singularities at Dirichlet/Neumann junctions) and prescribed high-order convergence for non-smooth domains.

A point of interest concerns the search algorithm in our eigensolver, which proceeds by searching for frequencies for which the integral equations of the problem admit non-trivial kernels. As it happens, the “minimum-singular-value” objective function gives rise to a challenging nonlinear optimization problem. To tackle this difficulty we put forth an improved objective functional which can be optimized by means of standard root-finding methods.

In addition, an integral-equation based methods for Steklov eigenvalue problem will be presented that exhibit spectral convergence for smooth domains (including multiply connected domains) and high order convergence for domains with corners and mixed boundary conditions (e.g. sloshing problem).

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Akhmetgaliyev, Eldar, Oscar Bruno, and Nilima Nigam, A boundary integral algorithm for the Laplace Dirichlet-Neumann mixed eigenvalue problem., arXiv preprint arXiv:1411.0071 (2014).