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"Discrete Ricci curvature with applications"

We define a notion of discrete Ricci curvature for a metric measure space by looking at whether "small balls are closer than their centers are". In a Riemannian manifolds this gives back usual Ricci curvature up to scaling. This definition is very easy to apply in a series of examples such as graphs (eg the discrete cube has positive curvature). We are able to generalize several Riemannian theorems in positive curvature, such as concentration of measure and the log-Sobolev inequality. This definition also allows to prove new theorems both in the Riemannian and discrete case: for example improved bounds on spectral gap of the Laplace-Beltrami operator, and fast convergence results for some Monte Carlo Markov Chain methods.

The plan of the minicourse is as follows:

- Visual introduction to curvatures of manifolds (too often books just give the formulas...)
- Discrete curvature using mass transport: definition, examples
- Properties of spaces with discrete positive curvature: spectral gap, concentration of measure
- Applications to Monte Carlo Markov chains simulation and statistics
- Applications to classical differential geometry: new spectral gap estimates from mass transport
- (time permitting) Link with combinatorics and Brunn-Minkowski inequality in the discrete hypercube