We shall consider complete non-compact Riemannian manifolds satisfying the volume doubling property. Heat kernel estimates have proven to be a powerful tool for analysis on such manifolds. In this framework, we will describe the relationship between several upper and lower bounds of the heat kernel, several types of functional inequalities, and the boundedness of the Riesz transform. We shall present old results, sometimes with new and more simple proofs, but also recent joint works with Adam Sikora and Salahaddine Boutayeb.

We shall treat a selection of the following topics :

-upper and lower, on-diagonal and off-diagonal heat kernel estimates, including the upper bound on the gradient, and their relationship : old and new

-characterization of the Gaussian upper estimate of the heat kernel in terms of one-parameter weighted inequalities

-the L^p boundedness of the Riesz transform : the case $1 < p < 2$ and the case $p > 2$

-a characterization of the gradient bound, and a sufficient condition for the boundedness of the Riesz transform, in terms of a Sobolev inequality, in the polynomial volume growth case.