Heat equation on some fractal metric spaces

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I will discuss the heat equation on some highly symmetric metric spaces with a fractal structure, such as the Sierpinski carpet. On these spaces analysis using upper derivatives does not seem to lead anywhere – for example it is not hard to prove that the usual type of Poincaré inequalities fail.

Instead, one needs some ‘renormalisation’; this yields a Dirichlet form $(\mathcal{E}, \mathcal{F})$; associated with this is a Lapalacian type operator $\mathcal{L}$ and solutions $p(t, x, y)$ to the heat equation

$$\frac{\partial u}{dt} = \mathcal{L}u.$$

These lectures will explore the properties of the Dirichlet form $(\mathcal{E}, \mathcal{F})$ and the solutions $p(t, x, y)$. An important role is played by a family of inequalities which control the energy of cut-off functions - i.e functions which are 1 on a ball $B(x, r)$ and zero on $B(x, 2r)^c$.

References

J. Kigami. Analysis on fractals. Cambridge University Press, 2001.

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