Abstract of Lectures at the
SÉMINAIRE DE MATHÉMATIQUES SUPÉRIEURES 2004
Morse theoretic methods in non-linear analysis and symplectic topology
Lectures on "Floer-homological methods in symplectic geometry"
by Matthias Schwarz
Specific Topic: Floer homology for the cotangent bundle

The aim of the 3 lectures is to present Floer homology in the context of the cotangent bundle for a class of Hamiltonians which have a suitable "quasi-quadratic" increase. A well-known result by Viterbo [Vit96] states that for the class of Hamiltonians equivalent to the geodesic flow on the cotangent bundle T^*M of a Riemannian manifold M, the associated Floer homology is isomorphic to the classical homology of the free loop space of M. A new joint work together with A. Abbondandolo [AS04] gives a new proof for this theorem extended by the statement that one obtains in fact a ring isomorphism between the pair-of-pants ring structure on Floer homology and the Chas-Sullivan loop product on the free loop space, [CS99]. These lectures are partially closely connected to the lectures given by A. Abbondandolo.

Lecture 1 The main results on the ring isomorphism will be presented. This requires the definitions and descriptions of the ring structures and homology theories involved. Both versions, for the based and for the free loop space will be given, involving the Hopf-algebra structure.

Lecture 2 will focus on details of the description of loop space homology in terms of infinite-dimensional Morse homology and on the precise construction of Floer homology on the cotangent bundle.

Lecture 3 will give the precise proof of the isomorphism using the approach of [AS04].

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