Homotopy theoretic tools in finite and infinite dimensional Morse theory.

OCTAV CORNEA - Montréal

There are two main ideas at the center of these talks: the first is that the moduli spaces of "connecting flow line type objects" in Morse (or Floer theory) are of considerable intrinsic interest; the second is that an efficient algebraic encoding of these spaces (of arbitrary dimensions) is provided by the differential of certain "rich" Morse-type complexes wich are obtained by an appropriate enlargement of the coefficient ring over which usual Morse complexes are defined.

A. In the first talk I will describe (roughly following [1]) a method to compactify in a natural way the unstable manifolds of a Morse-Smale function defined on a finite dimensional closed manifold M. This leads to a "rich" Morse type complex whose coefficients are in the cubical chains over the space of based loops on M and, as a consequence, I will show that the differentials of the Serre spectral sequence of the path-loop fibration of base M encode algebraically the moduli spaces of connecting (negative) gradient flow lines of arbitrary dimension. I will relate this result to Hopf invariant techniques introduced in [2] and [3] (with topological applications as in [4]) and to earlier work by Franks [6].

B. In the second talk these ideas will be shown to have effective applications to measuring algebraically high-dimensional moduli spaces of pseudoholomorphic strips relating Lagrangian manifolds when bubbling is not present. In particular, I will describe (as in [1]), how to easily deduce with these techniques a number of capacity type inequalitites and non-squeezing results. In this case an enriched Floer type complex is at the center of the story. The enlarged ring is, as in A., the cubical chains over a based loop space.

C. To appropriately encode bubbling, the ring at A. and B. is no longer sufficient and a new, essential enlargement is necessary. The last two talks will discuss how to proceed in this situation and will cover part of recent joint work with François Lalonde [5]. At the center of the construction is a new complex, called a clustered complex, whose role is to manage algebraically the bubbling of disks. Once this complex is defined and its algebraic properties are clarified it can be used as yet another enlarged coefficient ring to define an appropriate Morse-Floer type complex (called the fine Floer complex).

BIBLIOGRAPHIC REFERENCES

[1] Barraud, Jean-Francois and Cornea, Octav. Lagrangian intersections and the Serre spectral sequence, preprint. arXiv: math.DG/0401094.

[2] Cornea, Octavian. Homotopical dynamics. II. Hopf invariants, smoothings and the Morse complex. Ann. Sci. cole Norm. Sup. (4) 35 (2002), no. 4, 549-573.

[3] Cornea, Octavian. Homotopical dynamics. IV. Hopf invariants and Hamiltonian flows. Comm. Pure Appl. Math. 55 (2002), no. 8, 1033-1088.

[4] Cornea, Octavian. New obstructions to the thickening of CW-complexes. Proc. AMS posted March 205 (2004).

[5] Cornea, Octav and François Lalonde. A universal Floer homology, localizations and applications, in preparation.

[6] Franks, John. MorseSmale flows and homotopy theory, Topology 18 (1979) 199215.