Approximate Method for the Calculation of the Change in Impedance Due to a Flaw in a Conducting Cylindrical Layer*

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Abstract

Two approximate methods for the calculation of the change in impedance of a flawed cylindrical conducting tube are considered in the paper. The methods considered are the layer approximation and the Born approximation. The results of numerical computations of the change in impedance by means of the two methods are compared with experimental data. It is found that the layer approximation does not work well for cylindrical conducting layers.

Dedicated to the memory of Professor M. Ya. Antimirov.

Keywords: eddy current method, layer approximation, change in impedance

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Résumé

On considère la méthode des couches et celle de Born pour le calcul approché du changement d'impédance d'un cylindre conducteur avec paille. Les résultats numériques obtenus ne sont pas très près des données expérimentales.

1 Introduction

The aim of eddy current testing is to determine the parameters of a flaw contained in a conducting medium by analyzing the output signal from an eddy current probe. Several methods are available to solve this complicated inverse problem. The approach based on minimizing a cost function and using regularization algorithms is described in [1], [2]. Application of artificial neural networks to the solution of eddy current testing problems is discussed in [3]. Perturbation methods are often used in eddy current testing since the solution of physical problems related to the inspection of flaws in a conducting medium is either too complicated or time-consuming. Perturbation expansions are used in [4] and [5] to construct an approximate solution of eddy current testing problems in the case where the electrical conductivity of the flaw is close to that of the surrounding material.

A perturbation method, "the layer approximation", is described in [6]. The method is based on the assumption that that the flaw is not crack-like, that it is localized and that either the conducting material to be tested is layered or the relative change in conductivity introduced by the flaw is small. The layer approximation uses Auld's reciprocity theorem [7] and the solution [8], [9] for the vector potential in layered electrically conducting medium. The robustness of the layer approximation is tested in [6] where the flaw is modeled by a flat-bottom hole in an aluminium plate. It is found that the layer approximation is in a good agreement with experiments.

In the present paper, the layer approximation is used in a cylindrical conducting layer. Such problems arise in eddy current inspection of heat exchanger tubes [10]. The results are compared with experimental data. It is found that the layer approximation does not work well is such a configuration.

2 The cylindrical layer approximation

It is shown in [7] that the change in impedance of an eddy current probe can be written in the form

$$\Delta Z^{\text{ind}} = -\frac{1}{I^2} \iiint\limits_V \Delta\sigma \vec{E}_0 \cdot \vec{E}_{\prime f} dV \tag{1}$$

where $\Delta\sigma = \sigma - \sigma_0$ represents the difference in conductivity between the flawed sample and the reference sample, I is the current in the coil, V is the region where the flaw is located, $\vec{E}_0$ denotes the electric field in the reference part while $\vec{E}_f$denotes the electric field in the flawed part. Equation (1) depends on the electric field, $\vec{E}_f$, in the presence of the flaw (which is rarely known). Using the Born approximation one approximates the change in impedance by the formula

$$\Delta Z_B^{\text{ind}} \approx -\frac{1}{I^2} \iiint\limits_V \Delta\sigma \vec{E}_0 \cdot \vec{E}_{\prime 0} dV. \tag{2}$$

Under the layer approximation the change in impedance can be written in the form

$$\Delta Z_L^{\text{ind}} \approx -\frac{1}{I^2} \iiint\limits_V \Delta\sigma \vec{E}_0 \cdot \vec{E}_{\prime L} dV, \tag{3}$$

where the presence of the flaw is modeled by an additional hypothetical layer whose conductivity is equal to the conductivity of the flaw. In this case the electric field in the flawed sample is $\vec{E}_L$ which is calculated for this hypothetical layer.

The layer approximation was tested in [6] for the case of a flaw in the form of a cylindrical flat-bottom hole in a plate. The agreement between the experimental data and calculations based on formula (3) was found to be good.

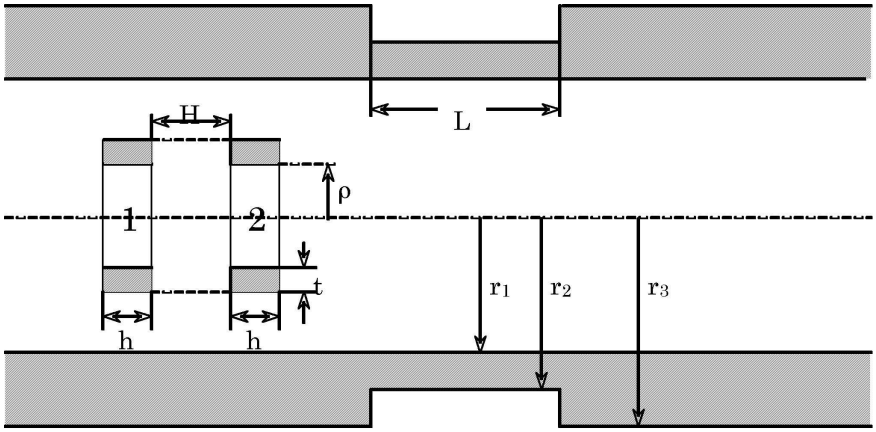

Figure 1: Metal tube with a flaw in the form of a 50% crack.

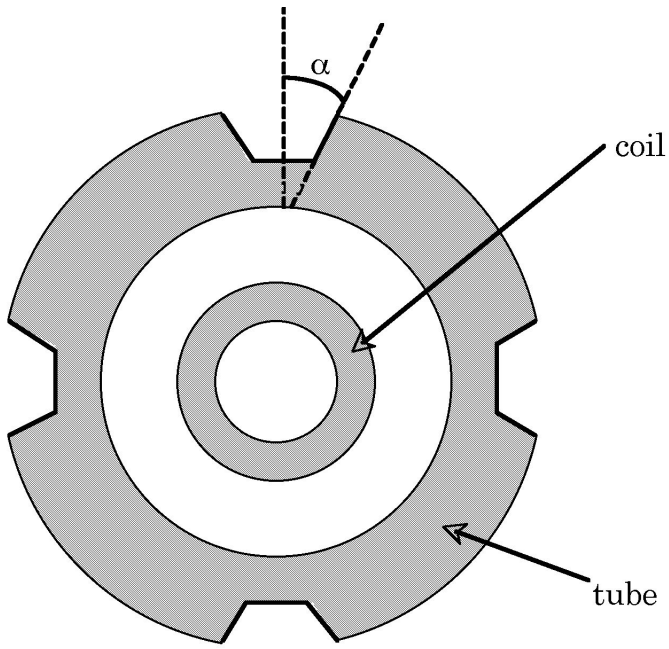

Figure 2: Cross-section of the tube with a flaw.

In the present paper we develop the layer approximation for a cylindrical metal layer. The formulas for the change in impedance of a coil with finite dimensions located inside the cylindrical layer are obtained in [9]. Thus, both $\vec{E}_0$ and $\vec{E}_L$ can be found from [9] and the integral in (3) can be calculated over the volume, V, of the flaw. The calculations are done for the configuration shown in Figs. 1 and 2.

The parameters are as follows: $r_1 = 9.85$ mm, $r_2 = 10.485$ mm, $r_3 = 11.12$ mm, $h = 2$ mm, $H = 2$ mm, $t = 2$ mm, $\rho = 7$ mm, $L = 20$ mm, the conductivity σ of the tube is $\sigma = 10^6$ S/m, each of the two coils (excitation coil 1 and test coil 2) have 60 turns, $\tan\alpha = 0.125/11.12$. The calculations were done for two values of the frequency, namely, $f = 100$ kHz and $f = 400$ kHz.

The change in impedance due to the flaw is shown in Figs. 3 and 4. The graphs show the layer approximation (solid line), the Born approximation (dashed line) and experimental data (dotted line). As can be seen from the graphs, the layer approximation does not work well in both cases. In addition, the Born approximation seems to be better than the layer approximation (at least it captures the shape of the impedance curve).

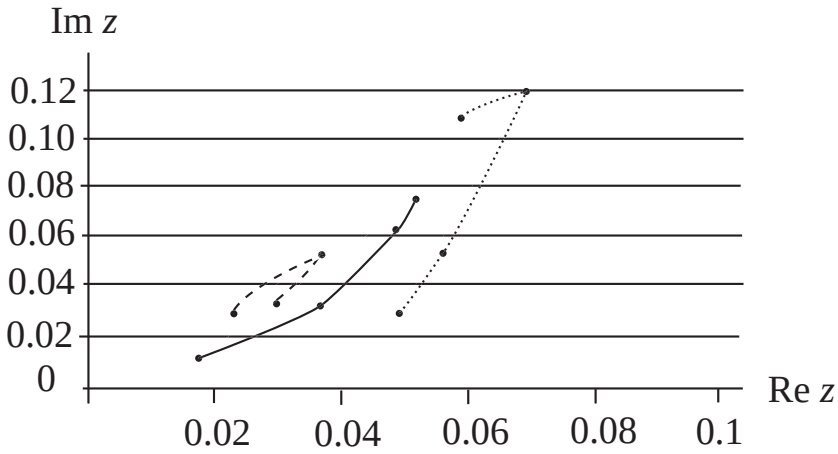

Figure 3: The change in impedance due to a flaw for the case $f = 100$kHz.

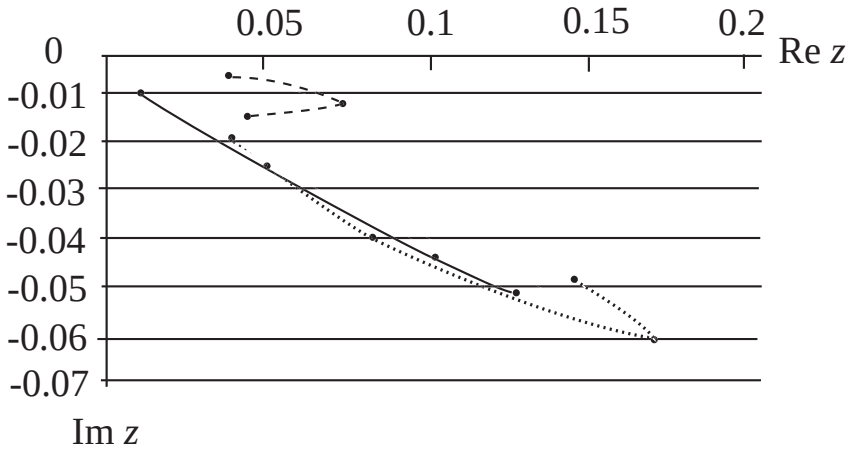

Figure 4: The change in impedance due to a flaw for the case $f = 400$kHz.

3 Conclusion

The layer approximation is used in the present paper to calculate the change in impedance of an eddy current probe due to a volumetric flaw in a cylindrical conducting tube. Calculations are compared with experimental data. It is shown that the layer approximation does not give satisfactory results in this case.

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