Tests for Non-Correlation of Two Multivariate Time Series: A Nonparametric Approach*

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Abstract

Most of the recent results on tests for non-correlation between two time series are based on the residual serial cross-correlation matrices resulting from appropriate modelling of the two series. However in the stationary case, test procedures can be defined from the serial cross-correlation of the original series, avoiding therefore the modelling stage. This paper aims at describing two such tests that take into account a finite number of lagged cross-correlations. The first one that is essentially valid for Gaussian time series makes use of a procedure for estimating the covariance structure of serial correlations described in Mélard, Paesmans and Roy (1991). The second one that is valid for a general class of mixing processes is based on the property that the cross-covariance at a given lag between two stationary processes is in fact the mean of the product of the two processes, the second one being lagged appropriately. For both approaches, the asymptotic distributions of the test statistics are derived under the null hypothesis of non-correlation between the two series. The level and power of the proposed tests are studied by simulation in finite samples and an example is presented.

Key words and phrases: stationarity; serial correlation; independence; spectral density; portmanteau test; causality.

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Résumé

La plupart des résultats récents sur les tests de non-corrélation entre deux séries chronologiques sont basés sur les matrices de corrélations croisées résiduelles résultant de la modélisation des deux séries. Cependant dans le cas stationnaire, des procédures de tests peuvent être développées directement à partir des corrélations croisées des séries originales, évitant ainsi la modélisation de chacune des deux séries. Le but de cet article est de décrire deux telles procédures qui prennent en compte les corrélations croisées à un nombre fini de délais. La première, qui est valide essentiellement pour des séries gaussiennes, fait intervenir la méthode d'estimation de la structure de covariance des corrélations sérielles décrite dans Mélard, Paesmans et Roy (1991). La deuxième qui est valide pour une classe générale de processus mélangeants s'appuie sur la propriété que la covariance croisée à un délai donné entre deux processus stationnaires est la moyenne du produit des deux processus, le deuxième étant décalé du délai correspondant. Dans les deux approches, les lois asymptotiques des statistiques de tests sont obtenues sous l'hypothèse nulle de non-corrélation entre les deux séries. Le niveau et la puissance des tests proposés sont étudiés à l'aide de simulation de Monte Carlo en échantillons finis et un exemple est présenté.

Mots-clés : stationnarité ; corrélation sérielle ; indépendance ; densité spectrale ; test portemanteau ; causalité.

1 Introduction

The existence of possible relationships between univariate or multivariate time series is a central question in many applications. Of particular interest in this context is the problem of testing non-correlation (or independence in the Gaussian case) between the observed series. It is therefore important to have methods which are simple both to apply and to interpret for checking non-correlation of two time series.

Most of the work done in this context is parametric and is based on the residuals of estimated models. In the case of two univariate time series $\{X^{(1)}(t)\}$ and $\{X^{(2)}(t)\}$, Haugh (1976) developed a procedure in which both series are supposed to be generated by stationary ARMA models. Non-correlation under such an assumption is equivalent to non-correlation of the two corresponding innovation processes. Denoting $\hat{a}^{(1)}(t)$ and $\hat{a}^{(2)}(t)$ the residuals resulting from fitting ARMA models to each of the two series separately and by $r_{\hat{a}}^{(12)}(k)$ the corresponding empirical cross-correlation at lag k, it is expected that information on possible linear relationships between the two series is contained in vectors of the form $\mathrm{r}_{\hat{a}}^{(12)} = (r_{\hat{a}}^{(12)}(-M), \ldots, r_{\hat{a}}^{(12)}(0), \ldots, r_{\hat{a}}^{(12)}(M))'$ where M is a fixed integer with respect to the length of the series. Under the null hypothesis of independence between the two original series $\{X^{(1)}(t)\}$ and $\{X^{(2)}(t)\}$, Haugh (1976) showed that $\sqrt{n}\mathrm{r}_{\hat{a}}^{(12)}$ asymptotically follows a multinormal distribution with mean zero and identity covariance matrix where n denotes the length of the series. This result leads to the definition of the portmanteau type statistic

$$Q_M = n \sum_{k=-M}^{M} [r_{\hat{a}}^{(12)}(k)]^2$$

which is asymptotically distributed as a χ^2_{2M+1} distribution and the hypothesis of non-correlation is rejected for large values of Q_M.

Haugh's procedure was extended in various directions. Koch and Yang (1986) introduced a modification of Q_M that allows for a potential pattern in the residual cross-correlation function. El Himdi and Roy (1997) proposed a version of Q_M for two stationary vector ARMA (VARMA) series that was recently extended to partially non stationary (cointegrated) VARMA series by Pham, Roy and Cédras (2001). Hallin and Saidi (2001) have recently developed a generalization of Koch and Yang procedure for VARMA series. For univariate time series, a robustified version of Haugh's statistic to outliers is described in Li and Hui (1994). Hong (1996) proposed a modification of Haugh's procedure for stationary infinite order autoregressive series in which a finite-order autoregression is fitted to each time series and the test statistic is a properly standardized version of the sum of weighted squared cross-correlations, with weights defined by a kernel function. A robustified version of Hong's statistic for univariate ARMA series is described in Duchesne and Roy (2001). Finally, Hallin *et al.* (1999) introduced a test for independence between two autoregressive time series, based on autoregressive rank scores.

The main objective of this paper is to develop two distinct nonparametric (model free) procedures for checking non-correlation of two multivariate stationary time series. The first is based on the consistent method described in Mélard, Paesmans and Roy (1991) for estimating the asymptotic covariance structure of empirical cross-correlations. The second one relies on the simple property that the cross-covariance at lag k between two stationary processes is in fact the expected value of the product of the two processes, the second one being lagged by k time intervals. This latter idea was exploited among others by Brillinger (1978).

In both approaches, test statistics at individual lags $\mathrm{QR}(k)$ and $\mathrm{QS}(k)$ respectively and the corresponding portmanteau type tests QR_M and QS_M that take into account all lags from $-M$ through $+M$ are described. The first approach relies on the rather strong assumption of linear processes whose fourth-order cumulants vanish whilst the second one is based on a mixing assumption extensively used in Brillinger (1975) which avoids the linearity assumption and the nullity of the fourth-order cumulants. For both approaches, the asymptotic distributions of the test statistics at individual lags and of the global statistics are derived under the null hypothesis.

The paper is organized as follows. In Section 2, we introduce the notations and we describe the asymptotic covariance structure of the serial cross-covariances and cross-correlations between two stationary processes. The approach based on the estimation of the asymptotic covariance structure of empirical cross-correlations is described in Section 3.1 and the second one based on the product of the two series is discussed in Section 3.2. When the null hypothesis of non-correlation is rejected, we usually want to identify the causality direction between the two series. A sufficient condition for non-causality in the sense of Granger (1969) is obtained in Section 4 and it suggests a descriptive causality analysis based on cross-correlations at positive (or negative) lags only. In Section 5, we report the result of a small simulation experiment to study the exact level of all proposed test statistics. Also, the powers of the two nonparametric portmanteau statistics are compared to the multivariate version of Haugh's test QH^*_M described in El Himdi and Roy (1997). It is seen that the loss of power resulting from the use of QR_M (with its exact critical values) in place of QH^*_M is rather small with the particular model considered. However, the test QS_M is considerably less powerful than QR_M for the series lengths considered ($n = 100, 200$). Finally, the procedure based on the statistics $\mathrm{QR}(k)$ and QR_M is applied to a set of economic data in Section 6.

2 Preliminaries

2.1 Definitions and notations

Let $\mathrm{X} = \{\mathrm{X}(t) = (X_1(t), \ldots, X_d(t))' : t \in \mathbb{Z}\}$ be a process with values in $\mathbb{R}^d$, second-order stationary whose mean is $\boldsymbol{\mu}_{\mathrm{X}}$. Let $\mathrm{E}[(\mathrm{X}(t) - \boldsymbol{\mu}_{\mathrm{X}})(\mathrm{X}(t-k) - \boldsymbol{\mu}_{\mathrm{X}})'] = \Gamma_{\mathrm{X}}(k)$, $t, k \in \mathbb{Z}$, be the autocovariance matrix at lag k. The autocorrelation matrix at lag k is given by

$$\boldsymbol{\rho}_{\mathrm{X}}(k) = (\rho_{ij}(k))_{d\times d}, \qquad \rho_{ij}(k) = \frac{\gamma_{ij}(k)}{\{\gamma_{ii}(0)\gamma_{jj}(0)\}^{1/2}} \ , \ 1 \le i, j \le d \ , \quad k \in \mathbb{Z},$$

where $\gamma_{ij}(k)$ is the $(i,j)-$element of $\Gamma_{\mathrm{X}}(k)$. Note that for any lag k, we have $\Gamma_{\mathrm{X}}(-k) = \Gamma_{\mathrm{X}}(k)'$ and $\boldsymbol{\rho}_{\mathrm{X}}(-k) = \boldsymbol{\rho}_{\mathrm{X}}(k)'$.

When the process X is partitioned into two subprocesses $\mathrm{X}^{(1)} = \{\mathrm{X}^{(1)}(t) : t \in \mathbb{Z}\}$ and $\mathrm{X}^{(2)} = \{\mathrm{X}^{(2)}(t) : t \in \mathbb{Z}\}$ of dimension d_1 and d_2 respectively with $d_1 + d_2 = d$, its autocovariance matrix $\Gamma_{\mathrm{X}}(k)$ can also be partitioned as

$$\Gamma_{\mathrm{X}}(k) = \begin{pmatrix} \Gamma_{\mathrm{X}}^{(11)}(k) & \Gamma_{\mathrm{X}}^{(12)}(k) \\ \Gamma_{\mathrm{X}}^{(21)}(k) & \Gamma_{\mathrm{X}}^{(22)}(k) \end{pmatrix}_{d\times d}, \quad k \in \mathbb{Z},$$

where the two diagonal blocks $\Gamma_{\mathrm{X}}^{(11)}(k)$ and $\Gamma_{\mathrm{X}}^{(22)}(k)$ represent the autocovariance matrices at lag k of the processes $\mathrm{X}^{(1)}$ and $\mathrm{X}^{(2)}$ respectively, $\Gamma_{\mathrm{X}}^{(12)}(k)$ is the cross-covariance matrix at lag k between the two subprocesses and $\Gamma_{\mathrm{X}}^{(21)}(k) = \Gamma_{\mathrm{X}}^{(12)}(-k)'$. The autocorrelation matrix $\boldsymbol{\rho}_{\mathrm{X}}(k)$ can be partitioned similarly in $\boldsymbol{\rho}_{\mathrm{X}}^{(11)}(k), \boldsymbol{\rho}_{\mathrm{X}}^{(22)}(k)$ and $\boldsymbol{\rho}_{\mathrm{X}}^{(12)}(k)$.

Given a realization $\mathrm{X}(1), \ldots, \mathrm{X}(n)$ of length n of the process X, the sample autocovariance matrix at lag k, $(1 \le k \le n-1)$, is defined by $\mathrm{C}_{\mathrm{X}}(k) = n^{-1}\sum_{t=k+1}^{n}(\mathrm{X}(t) - \bar{\mathrm{X}})(\mathrm{X}(t-k) - \bar{\mathrm{X}})'$, where $\bar{\mathrm{X}} = n^{-1}\sum_{t=1}^{n}\mathrm{X}(t) = (\bar{X}_1, \ldots, \bar{X}_d)'$ is the sample mean. We let $\mathrm{C}_{\mathrm{X}}(k) = \mathrm{C}_{\mathrm{X}}(-k)'$ for $1-n \le k \le 0$ and $\mathrm{C}_{\mathrm{X}}(k) = 0$ for $|k| \ge n$. The sample autocorrelation matrix at lag k, $0 \le |k| \le n-1$, is defined by

$$\mathrm{R}_{\mathrm{X}}(k) = (r_{ij}(k))_{d\times d} \ , \quad r_{ij}(k) = \frac{c_{ij}(k)}{\{c_{ii}(0)c_{jj}(0)\}^{1/2}} \ , \tag{2.1}$$

where $c_{ij}(k)$ is the $(i,j)-$element of $\mathrm{C}_{\mathrm{X}}(k)$. Equation (2.1) can be equivalently written as $\mathrm{R}_{\mathrm{X}}(k) = \mathrm{D}\{c_{ii}^{-1/2}(0)\}\mathrm{C}_{\mathrm{X}}(k)\mathrm{D}\{c_{jj}^{-1/2}(0)\}$ where $\mathrm{D}\{a_i\}$ is the diagonal matrix of dimension d with diagonal elements given by $a_1, \ldots, a_d$. When $\mathrm{X}(t)$ is partitioned into two sub-vectors as before, the matrix $\mathrm{R}_{\mathrm{X}}(k)$ is partitioned in a similar manner to $\Gamma_{\mathrm{X}}(k)$. In particular, the cross-correlation matrix at lag k between the two time series $\{\mathrm{X}^{(1)}(t)\}$ and $\{\mathrm{X}^{(2)}(t)\}$ is defined by

$$\mathrm{R}_{\mathrm{X}}^{(12)}(k) = \mathrm{D}\{(c_{ii}^{(11)}(0))^{-1/2}\}\mathrm{C}_{\mathrm{X}}^{(12)}(k)\mathrm{D}\{(c_{jj}^{(22)}(0))^{-1/2}\}, \tag{2.2}$$

with

$$\mathrm{C}_{\mathrm{X}}^{(hl)}(k) = \frac{1}{n}\sum_{t=k+1}^{n}\{\mathrm{X}^{(h)}(t) - \bar{\mathrm{X}}^{(h)}\}\{\mathrm{X}^{(l)}(t-k) - \bar{\mathrm{X}}^{(l)}\}' = (c_{ij}^{(hl)}(k))_{d_h\times d_l},$$

$\bar{\mathrm{X}}^{(h)}$ being the sample means of the time series $\mathrm{X}^{(h)}(t)$, $h = 1, 2$.

Let $k_1, \ldots, k_m$ a sequence of m arbitrary distinct integers independent of n and such that $k_i < n$. We define the vectors $\boldsymbol{\rho}_{\mathrm{X}}$ and r_{X} of theoretical and sample autocorrelations of dimension md^2 by

$$\boldsymbol{\rho}_{\mathrm{X}} = (\mathrm{vec}\boldsymbol{\rho}_{\mathrm{X}}(k_1)', \ldots, \mathrm{vec}\boldsymbol{\rho}_{\mathrm{X}}(k_m)')', \qquad \mathrm{r}_{\mathrm{X}} = (\mathrm{vec}\mathrm{R}_{\mathrm{X}}(k_1)', \ldots, \mathrm{vec}\mathrm{R}_{\mathrm{X}}(k_m)')', \tag{2.3}$$

where 'vec' stands for the usual matrix operator that transforms a matrix into a vector by stacking its columns. Similarly, the vectors γ_{X} and c_{X} of theoretical and sample autocovariances are given by

$$\gamma_{\mathrm{X}} = (\mathrm{vec}\Gamma_{\mathrm{X}}(k_1)', \ldots, \mathrm{vec}\Gamma_{\mathrm{X}}(k_m)')', \qquad \mathrm{c}_{\mathrm{X}} = (\mathrm{vec}\mathrm{C}_{\mathrm{X}}(k_1)', \ldots, \mathrm{vec}\mathrm{C}_{\mathrm{X}}(k_m)')'. \tag{2.4}$$

The vectors $\mathrm{r}_{\mathrm{X}}^{(12)}$ and $\mathrm{c}_{\mathrm{X}}^{(12)}$ of sample cross-correlations and cross-covariances of dimension md_1d_2 are defined by

$$\mathrm{r}_{\mathrm{X}}^{(12)} = (\mathrm{vec}\mathrm{R}_{\mathrm{X}}^{(12)}(k_1)', \ldots, \mathrm{vec}\mathrm{R}_{\mathrm{X}}^{(12)}(k_m)')', \tag{2.5}$$

$$\mathrm{c}_{\mathrm{X}}^{(12)} = (\mathrm{vec}\mathrm{C}_{\mathrm{X}}^{(12)}(k_1)', \ldots, \mathrm{vec}\mathrm{C}_{\mathrm{X}}^{(12)}(k_m)')'. \tag{2.6}$$

With an adequate choice of the lags $k_1, \ldots, k_m$, one or the other of these vectors allows us to detect most of the relationships existing between the two series. Their asymptotic distributions can be directly deduced from those of r_{X} and c_{X}.

2.2 Asymptotic distribution of serial correlations and covariances

By the Wold decomposition, the purely non-deterministic and zero-mean process X admits the representation

$$\mathrm{X}(t) = \sum_{j=0}^{\infty} \Psi_j \mathrm{a}(t-j), \qquad t \in \mathbb{Z},$$

where the Ψ_j are $d \times d$ matrices such that $\sum_{j=0}^{\infty} ||\Psi_j||^2 < \infty$, $||\cdot||$ being the euclidian norm, $\Psi_0 = \mathrm{I}$ is the identity matrix of dimension d and the innovation process $\{\mathrm{a}(t) : t \in \mathbb{Z}\}$ is a weak white noise, that is the vectors $\mathrm{a}(t)$ are uncorrelated with mean 0 and with regular covariance matrix Ω. Let F_t be the sub $\sigma-$algebra (of the $\sigma-$algebra with respect to which all the components $X_i(t)$ are measurable) generated by $X_i(s), s \leq t, i = 1, \ldots, d$. The following assumption was made by Hannan (1976) and by Roy (1989) in the study of the asymptotic distribution of serial correlations and covariances.

Assumption A1. Suppose that $\mathrm{E}[a_i(t)|F_{t-1}]$, $\mathrm{E}[a_i(t)a_j(t)|F_{t-1}]$, $\mathrm{E}[a_i(t)a_j(t)a_l(t)|F_{t-1}]$ and $\mathrm{E}[a_i(t)a_j(t)a_l(t)a_m(t)|F_{t-1}]$ exist and are constant (with respect to t) for all i, j, l, m.

This assumption is satisfied if the $\mathrm{a}(t)$ are independent and identically distributed. In addition, let us write $\kappa_{ijkm} = \text{cumulant}\{a_i(t), a_j(t), a_k(t), a_m(t)\}$. Let the symbol '$\xrightarrow{L}$' stands for 'convergence in law'. It follows from Roy (1989) that under Assumption A.1, if all fourth-order cumulants κ_{ijkm} are zero and if the spectral density of each component of $\mathrm{X}(t)$ is square integrable then

$$\sqrt{n}(\mathrm{r_X} - \boldsymbol{\rho}_\mathrm{X}) \xrightarrow{L} \mathrm{N}(0, \Sigma),$$

where $\mathrm{r_X}$ and $\boldsymbol{\rho}_\mathrm{X}$ are the vectors of correlations defined by (2.3) and Σ is the asymptotic covariance matrix whose elements are given by relation (5) in Roy (1989). The asymptotic normality of the vector $\mathrm{r}_\mathrm{X}^{(12)}$ of cross-correlations defined by (2.5) is an immediate consequence. Its asymptotic covariance matrix $\Sigma = (\Sigma_{k_i,k_j})_{1 \leq i,j \leq m}$ is a block matrix of dimension md_1d_2 where each block Σ_{k_i,k_j} represents the asymptotic covariance matrix between the vectors $\mathrm{vecR}_\mathrm{X}^{(12)}(k_i)$ and $\mathrm{vecR}_\mathrm{X}^{(12)}(k_j)$. Under the same assumptions, we have from Hannan (1976) that the asymptotic distribution of the vector $\mathrm{c_X}$ defined in (2.4) is the following

$$\sqrt{n}(\mathrm{c_X} - \gamma_\mathrm{X}) \xrightarrow{L} \mathrm{N}(0, \tilde{\Sigma}),$$

where the asymptotic covariance matrix $\tilde{\Sigma}$ is obtained from relation (5) of Hannan (1976). The asymptotic normality of the cross-covariance vector $\mathrm{c}_\mathrm{X}^{(12)}$ is characterized by the asymptotic covariance matrix $\tilde{\Sigma} = (\tilde{\Sigma}_{k_i,k_j})_{1 \leq i,j \leq m}$ of dimension md_1d_2 in which the block $\tilde{\Sigma}_{k_i,k_j}$ represents the asymptotic covariance matrix between $\mathrm{vecC}_\mathrm{X}^{(12)}(k_i)$ and $\mathrm{vecC}_\mathrm{X}^{(12)}(k_j)$.

Under the null hypothesis of non-correlation $H_0 : \boldsymbol{\rho}_\mathrm{X}^{(12)}(k) = 0$, for all $k \in \mathbb{Z}$, the matrices Σ_{k_i,k_j} and $\tilde{\Sigma}_{k_i,k_j}$ take a simple form. Under H_0, only the last term in relation (5) of Roy (1989) is different from zero and we have

$$\Sigma_{k_i,k_j} = \sum_{u=-\infty}^{+\infty} \boldsymbol{\rho}_\mathrm{X}^{(22)}(u + k_j - k_i) \otimes \boldsymbol{\rho}_\mathrm{X}^{(11)}(u). \tag{2.7}$$

Therefore, Σ_{k_i,k_j} depends only on the difference of the two lags k_i and k_j and we will write in the sequel $\Sigma_{k,h} = \Sigma_{h-k}$, where (with $l = h - k$)

$$\Sigma_l = \sum_{u=-\infty}^{+\infty} \boldsymbol{\rho}_\mathrm{X}^{(22)}(u + l) \otimes \boldsymbol{\rho}_\mathrm{X}^{(11)}(u). \tag{2.8}$$

In the same manner, under the previous assumptions and under H_0, relation (5) in Hannan (1976) takes a much simpler form and we can write $\tilde{\Sigma}_{k_i,k_j} = \tilde{\Sigma}_{k_j-k_i}$ where

$$\tilde{\Sigma}_l = \sum_{u=-\infty}^{+\infty} \Gamma_\mathrm{X}^{(22)}(u + l) \otimes \Gamma_\mathrm{X}^{(11)}(u). \tag{2.9}$$

We must note that the asymptotic normality of the vectors of covariances or correlations is established under the assumption that all fourth-order cumulants of the process $\mathrm{X} = (\mathrm{X}^{(1)'}, \mathrm{X}^{(2)'})'$ vanish and that assumption is satisfied for Gaussian processes. If the fourth-order cumulants are non zero, the asymptotic normality of the vectors $\mathrm{r_X}$ and $\mathrm{r}_\mathrm{X}^{(12)}$ still holds. However, the asymptotic covariance structure is more complex since it also depends on these cumulants, see Hannan (1976). Berlinet and Francq (1997, 1999) studied the estimation of the asymptotic covariance structure under the weaker assumption of strongly mixing processes.

3 Nonparametric tests for independence

The vectors of sample autocorrelations and autocovariances, $\text{vecR}_{\text{X}}^{(12)}(k)$ and $\text{vecC}_{\text{X}}^{(12)}(k)$, can be used to test the hypothesis of non-correlation between two multivariate processes $\text{X}^{(1)}$ and $\text{X}^{(2)}$. Under Assumption A.1 and the null hypothesis of non-correlation, we have from Section 2.2 that

$$\sqrt{n}\text{vecR}_{\text{X}}^{(12)}(k) \xrightarrow{L} \text{N}(0, \Sigma_0), \tag{3.1}$$

where Σ_0 is given by (2.8), and

$$\sqrt{n}\text{vecC}_{\text{X}}^{(12)}(k) \xrightarrow{L} \text{N}(0, \tilde{\Sigma}_0), \tag{3.2}$$

where $\tilde{\Sigma}_0$ is given by (2.9).

>From a multivariate version of Proposition 5.1.1 of Brockwell and Davis (1991), it follows that Σ_0 is positive definite if $\Gamma_{\text{X}}^{(jj)}(0)$ is regular and if $\Gamma_{\text{X}}^{(jj)}(0) \to 0$ as $h \to \infty$, $j = 1, 2$. The estimation of the asymptotic covariance matrices Σ_0 and $\tilde{\Sigma}_0$ allows us to deduce a simple and natural procedure for testing H_0 against the alternative $H_{1k} \ : \ \boldsymbol{\rho}_{\text{X}}^{(12)}(k) \neq 0$.

Another nonparametric approach for inference on the theoretical autocorrelations $\boldsymbol{\rho}_{\text{X}}^{(12)}(k)$ when the fourth-order cumulants of the joint process X are non zero is also presented. It is based on the property that the cross-covariance at a given lag between two stationary processes is in fact the mean of the product of the two processes, the second one being lagged appropriately. This idea was exploited among others by Brillinger (1978).

3.1 Approach based on the estimation of the asymptotic covariance

The method of Mélard *et al.* (1991) allows us to construct consistent estimators of the covariance matrices $\Sigma = (\Sigma_{k-h})$ and $\tilde{\Sigma} = (\tilde{\Sigma}_{k-h})$ noted $\hat{\Sigma} = (\hat{\Sigma}_{k-h})$ and $\hat{\tilde{\Sigma}} = (\hat{\tilde{\Sigma}}_{k-h})$. Under H_0, $\hat{\Sigma}_l$ when $l = k - h$ is given by

$$\hat{\Sigma}_l = \sum_{u=-T_n}^{T_n - l} w((u+l)/T_n)\text{R}_{\text{X}}^{(22)}(u+l) \otimes w(u/T_n)\text{R}_{\text{X}}^{(11)}(u), \tag{3.3}$$

for $l \geq 0$ and $\hat{\Sigma}_l = \hat{\Sigma}_{-l}'$ for $l < 0$. In expression (3.3), $w : \mathbb{R} \to \mathbb{R}$ is a positive definite symmetric weight function (also called a kernel), which is continuous at zero, $w(0) = 1$, has at most a finite number of discontinuity points, is bounded and square integrable. The sequence $\{T_n\}$ of real numbers is such that $T_n \to \infty$ but not too fast, that is $n/T_n \to \infty$ when $n \to \infty$. Under the assumption that the sample covariances $\text{C}_{\text{X}}^{(jj)}(0)$, $j = 1, 2$, are positive definite, it follows from the multivariate version of problem 7.11 of Brockwell and Davis (1991) that $\hat{\Sigma} = (\hat{\Sigma}_{k-h})$ is also positive definite. In the numerical illustration presented, Mélard *et al.* (1991) used the modified Bartlett and Parzen windows which are positive definite functions and the truncation points $T_n = H\sqrt{n}, H = 1, 3, 5$. An analogous estimator $\hat{\tilde{\Sigma}}_h$ for $\tilde{\Sigma}_h$ is obtained by replacing in (3.3) the autocorrelations $\text{R}_{\text{X}}^{(hh)}(k)$ by the autocovariances $\text{C}_{\text{X}}^{(hh)}(k)$, $h = 1, 2$.

3.1.1 Tests based on the correlations

Let M be a fixed positive integer independent of n with $M < n - 1$. For each k such that $0 \leq |k| \leq M$, we define the statistic $\text{QR}(k)$ by

$$\text{QR}(k) = n\text{vecR}_{\text{X}}^{(12)}(k)' \ \hat{\Sigma}_0^{-1} \ \text{vecR}_{\text{X}}^{(12)}(k). \tag{3.4}$$

According to (3.1) and since the estimator $\hat{\Sigma}_0$ defined by (3.3) is consistent, the statistic $\text{QR}(k)$ is asymptotically distributed under the null hypothesis H_0 as a chi square variable with $d_1 d_2$ degrees of freedom. The test procedure based on $\text{R}_{\text{X}}^{(12)}(k)$ to test H_0 against the alternative H_{1k} is the following.

Given two realizations $\text{X}^{(1)}(1), \ldots, \text{X}^{(1)}(n)$ and $\text{X}^{(2)}(1), \ldots, \text{X}^{(2)}(n)$:

Step 1 For $|u| \leq T_n$, compute the autocorrelations $\text{R}_{\text{X}}^{(11)}(u), \text{R}_{\text{X}}^{(22)}(u)$ defined by (2.1) and the cross-correlations $\text{R}_{\text{X}}^{(12)}(u)$ defined by (2.2).

Step 2 Compute the matrix $\hat{\Sigma}_0$ defined by (3.3) and the statistic $\text{QR}(k)$ defined by (3.4).

Step 3 For a given significance level α, reject H_0 if $\text{QR}(k) > \chi^2_{d_1 d_2, 1-\alpha}$, where $\chi^2_{m,\alpha}$ is the αth quantile of the chi square distribution with m degrees of freedom.

This procedure can also be formulated in terms of the sample cross-covariances with the statistic QC(k) defined by

$$\mathrm{QC}(k) = n \mathrm{vecC}_{\mathrm{X}}^{(12)}(k)' \, \hat{\tilde{\Sigma}}_0^{-1} \, \mathrm{vecC}_{\mathrm{X}}^{(12)}(k), \tag{3.5}$$

where $\hat{\tilde{\Sigma}}_0$ is the estimator defined as in (3.3) of the asymptotic covariance matrix $\tilde{\Sigma}_0$ of (3.2). In our context, the two statistics QR(k) and QC(k) are in fact identical.

Lemma 3.1

$$QR(k) = QC(k), \quad |k| \leq M.$$

Proof : We have $\mathrm{R}_{\mathrm{X}}^{(hh)}(k) = \mathrm{D}_h \mathrm{C}_{\mathrm{X}}^{(hh)}(k) \mathrm{D}_h, \ h = 1, 2$, where $\mathrm{D}_h = Diag\{(c_{jj}^{(hh)}(0))^{-1/2}\}$. We can also write using the property $(\mathrm{A} \otimes \mathrm{B})(\mathrm{C} \otimes \mathrm{D}) = (\mathrm{AC}) \otimes (\mathrm{BD})$ of the Kronecker product,

$$\begin{aligned} \hat{\Sigma}_0 &= \sum_{u=-T_n}^{T_n} \{w(u/T_n) \mathrm{D}_2 \mathrm{C}_{\mathrm{X}}^{(22)}(u) \mathrm{D}_2\} \otimes \{w(u/T_n) \mathrm{D}_1 \mathrm{C}_{\mathrm{X}}^{(11)}(u) \mathrm{D}_1\} \\ &= (\mathrm{D}_2 \otimes \mathrm{D}_1) \hat{\tilde{\Sigma}}_0 (\mathrm{D}_2 \otimes \mathrm{D}_1). \end{aligned} \tag{3.6}$$

Replacing in (3.4) the matrix $\hat{\Sigma}_0$ by its expression (3.6) and using the relation $\mathrm{vecR}_{\mathrm{X}}^{(12)}(k) = (\mathrm{D}_2 \otimes \mathrm{D}_1) \mathrm{vecC}_{\mathrm{X}}^{(12)}(k)$, the proof is completed.

As in Haugh (1976) and in El Himdi and Roy (1997), the modified statistics

$$\mathrm{QR}^*(k) = \frac{n}{n - |k|} \mathrm{QR}(k) \tag{3.7}$$

were also considered.

In practice, we usually want to simultaneously take into account many lags, and if no particular direction of causality is assumed a priori, the set of lags such that $|k| \leq M$, where $M < n - 1$ is fixed with respect to n, is a reasonable choice. Thus for the alternative hypothesis $H_1^{(M)}$:

$$H_1^{(M)} \quad : \text{There exists at least one } k, |k| \leq M, \text{for which } \boldsymbol{\rho}_{\mathrm{X}}^{(12)}(k) \neq 0, \tag{3.8}$$

a global test of level at most α consists of rejecting H_0 if for at least one lag k, $\mathrm{QR}(k) > \chi^2_{d_1 d_2, 1-\alpha_M}$, where $\alpha_M = \alpha/(2M+1)$, invoking the Bonferroni inequality.

We can also consider a portmanteau type test for H_0 against $H_1^{(M)}$ whose asymptotic level is exactly α. According Section 2.2, the covariance structure under H_0 of the vector $\mathrm{r}_{\mathrm{X}}^{(12)} = (\mathrm{vec}R_{\mathrm{X}}^{(12)}(-M)', \ldots, \mathrm{vec}R_{\mathrm{X}}^{(12)}(M)')'$, is known and we can define the following statistic

$$\mathrm{QR}_M = n \mathrm{r}_{\mathrm{X}}^{(12)'} \hat{\Sigma}^{-1} \mathrm{r}_{\mathrm{X}}^{(12)}, \tag{3.9}$$

where $\hat{\Sigma} = (\hat{\Sigma}_{k-h})_{0 \leq |k|, |h| \leq M}$ and $\hat{\Sigma}_l$ is given by (3.3). The statistic QR_M is asymptotically distributed as a chi square variable with $(2M+1)d_1 d_2$ degrees of freedom when $\hat{\Sigma}$ is of full rank. If not, $\hat{\Sigma}^{-1}$ is replaced in (3.9) by a generalized inverse of $\hat{\Sigma}$, and the number of degrees of freedom corresponds to $\mathrm{rank}(\hat{\Sigma}_M) d_1 d_2$, see for example Rao and Mitra (1971, p. 173). We can also introduce an analogous statistic QC_M, based on the vector of autocovariances $\mathrm{c}_{\mathrm{X}}^{(12)} = (\mathrm{vec}C_{\mathrm{X}}^{(12)}(-M)', \ldots, \mathrm{vec}C_{\mathrm{X}}^{(12)}(M)')'$, and its estimated asymptotic covariance matrix $\hat{\tilde{\Sigma}}$. That statistic is defined by

$$\mathrm{QC}_M = n \ \mathrm{c}_{\mathrm{X}}^{(12)'} \hat{\tilde{\Sigma}}^{-1} \mathrm{c}_{\mathrm{X}}^{(12)}. \tag{3.10}$$

Whilst $\mathrm{QR}(k) = \mathrm{QC}(k)$ by Lemma 3.1, the two statistics QR_M and QC_M are in general different.

3.2 Approach based on the estimation of the variance of a mean

Let us assume that the d vector-valued process X is strictly stationary, that its moments of all orders exist and that its mean is 0. Its joint cumulant function of order k is defined by

$$\kappa_{a_1, \ldots, a_k}(t_1, \ldots, t_{k-1}) = \ \mathrm{cumulant}\{X_{a_1}(t_1 + \tau), \ldots, X_{a_{k-1}}(t_{k-1} + \tau), X_{a_k}(\tau)\},$$

$a_1, \ldots, a_k \in \{1, \ldots, d\}, \ t_1, \ldots, t_{k-1}, \tau \in \mathbb{Z}, k = 2, 3, \ldots$. As in Brillinger (1975), we will suppose that the span of dependence of X is small enough as described by the following assumption.

Assumption A2: The multivariate process X is stricly stationary, its moments of all orders exist and are such that its cumulant function of order k is absolutely summable, that is

$$\sum_{t_1,\dots,t_{k-1}=-\infty}^{+\infty} |\kappa_{a_1,\dots,a_k}(t_1,\dots,t_{k-1})| < \infty \quad , \quad a_1,\dots,a_k \in \{1,\dots,d\},\ k=2,3,\dots \tag{3.11}$$

As before, let us write $\mathrm{X} = (\mathrm{X}^{(1)'}, \mathrm{X}^{(2)'})'$. For a fixed integer k, let $\mathrm{Y}_k = \{\mathrm{Y}_k(t) : t \in \mathbb{Z}\}$ be the process of dimension $q = d_1 d_2$ defined by

$$\mathrm{Y}_k(t) = \mathrm{X}^{(1)}(t) \otimes \mathrm{X}^{(2)}(t-k) \quad , \quad t \in \mathbb{Z}. \tag{3.12}$$

The stationarity of X implies that the transformed process Y_k is also stationary and that its mean is given by

$$\boldsymbol{\mu}_Y(k) = E[\mathrm{Y}_k] = \mathrm{vec}\{\Gamma_{\mathrm{X}}^{(12)}(k)'\}'. \tag{3.13}$$

Therefore, testing that $\Gamma_{\mathrm{X}}^{(12)}(k) = 0$ is equivalent to testing that $\boldsymbol{\mu}_Y(k) = 0$. Brillinger (1978) exploited that property for developing confidence intervals for the cross-covariance function of two univariate time series. The following result is a direct consequence of the multivariate version of Theorem 2.9.1 of Brillinger (1975).

Proposition 3.1 *If the process* X *satisfies Assumption A.2, then the transformed process* Y_k *defined by (3.12) also satisfies A.2.*

>From (3.13), $\boldsymbol{\mu}_Y(k) = 0$ is equivalent to $\Gamma_{\mathrm{X}}^{(12)}(k) = 0$, and classic results on the estimation of the mean of a stationary multivariate time series can be used in order to derive tests for non-correlation.

Let $\mathrm{Y} = \{\mathrm{Y}(t) : t \in \mathbb{Z}\}$ be a q-dimensional process satisfying A.2 with mean $\boldsymbol{\mu}_Y$ and spectral density $\mathrm{f}_Y(\cdot)$. From a realization $\mathrm{Y}(1), \dots, \mathrm{Y}(n)$ of length n, the periodogram at frequency $\lambda \in \mathbb{R}$ is defined by

$$\mathrm{I}_Y^{(n)}(\lambda) = \frac{1}{2\pi n} \mathrm{d}_Y^{(n)}(\lambda) \overline{\mathrm{d}_Y^{(n)}(\lambda)'}$$

where $\bar{z}$ denotes the conjugate of the complex number z and $\mathrm{d}_Y^{(n)}(\lambda)$ represents the finite Fourier transform of the series:

$$\mathrm{d}_Y^{(n)}(\lambda) = \sum_{t=1}^{n} \mathrm{Y}(t) e^{-i\lambda(t-1)}.$$

The arithmetic mean $\bar{\mathrm{Y}}$ of the realization $\mathrm{Y}(1), \dots, \mathrm{Y}(n)$ can be written as $\bar{\mathrm{Y}} = n^{-1}\mathrm{d}_Y^{(n)}(0)$ and from Theorem 4.4.1 of Brillinger (1975), we have that

$$\sqrt{n}\bar{\mathrm{Y}} \xrightarrow{L} \mathrm{N}_q(\boldsymbol{\mu}_Y, 2\pi \mathrm{f}_Y(0)), \tag{3.14}$$

A test statistic for $H_0 : \boldsymbol{\mu}_Y = 0$ is obtained from (3.14) by replacing $\mathrm{f}_Y(0)$ by a consistent estimator. Several consistent estimators of $\mathrm{f}_Y(0)$ are available; see for example Brillinger (1975, Chap. 7). Here, we will use the smoothed periodogram defined by

$$\mathrm{f}_Y^{(n)}(0) = \frac{1}{2J} \sum_{s=1}^{J} \{\mathrm{I}_Y^{(n)}(\frac{2\pi s}{n}) + \mathrm{I}_Y^{(n)}(\frac{-2\pi s}{n})\} = \frac{1}{J} \sum_{s=1}^{J} \mathrm{Re}\{\mathrm{I}_Y^{(n)}(\frac{2\pi s}{n})\}, \tag{3.15}$$

where $1 \le J < n/2$ is a positive integer and Re{A} denotes the real part of the complex-valued matrix A. The joint asymptotic distribution of $\bar{\mathrm{Y}}$ and $\mathrm{f}_Y^{(n)}(0)$ is given by the following proposition.

Proposition 3.2 *Let* $\{\mathrm{Y}(t) : t \in \mathbb{Z}\}$ *be a process satisfying Assumption A.2. Then the matrices* $\sqrt{n}(\bar{\mathrm{Y}} - \boldsymbol{\mu}_Y)$ *and* $\mathrm{f}_Y^{(n)}(0)$ *jointly converge in distribution, as* $n \to \infty$, *to* U_0 *and* U_1, *where* U_0 *and* U_1 *are independent,* U_0 *follows a* $N_q(0, 2\pi \mathrm{f}_Y(0))$ *distribution and* U_1 *is a random matrix distributed as* $\frac{1}{2J}\mathrm{W}_q(2J, \mathrm{f}_Y(0))$, *where* $\mathrm{W}_q(m, \Omega)$ *represents the real Wishart distribution with* m *degrees of freedom and of fundamental matrix* Ω.

Proof : A direct consequence of Theorem 4.4.1 of Brillinger (1975) is that the $J+1$ random matrices $n^{1/2}\mathrm{d}_Y^{(n)}(0)$, $n^{1/2}\mathrm{I}_Y^{(n)}(\frac{2\pi}{n}), \dots, n^{1/2}\mathrm{I}_Y^{(n)}(\frac{2\pi J}{n})$, where J is a positive integer independent of n, converge in distribution to random independent matrices $\mathrm{U}_0, \mathrm{U}_1, \dots, \mathrm{U}_J$, where U_0 is $\mathrm{N}_q(\boldsymbol{\mu}_Y, 2\pi \mathrm{f}_Y(0))$ and U_j, $j = 1, \dots J$, are random $\mathrm{W}_q^{\mathcal{C}}(1, \mathrm{f}_Y(0))$ matrices. The symbol $\mathrm{W}_q^{\mathcal{C}}(1, \mathrm{f}_Y(0))$ stands for the complex Wishart distribution with one degree of freedom and fundamental matrix $\mathrm{f}_Y(0)$. The stated result follows from (3.14) and the additive property of independent random Wishart matrices. The proof is completed.

If $\mathrm{f}_\mathrm{Y}(0)$ is regular, $\mathrm{f}_Y^{(n)}(0)$ is also regular for n sufficiently large and the following statistic

$$S_n = \frac{n}{2\pi}(\bar{\mathrm{Y}} - \boldsymbol{\mu}_Y)^{'}\{\mathrm{f}_Y^{(n)}(0)\}^{-1}(\bar{\mathrm{Y}} - \boldsymbol{\mu}_Y) \tag{3.16}$$

is well defined. The joint convergence in distribution of $\bar{\mathrm{Y}}$ and $\mathrm{f}_Y^{(n)}(0)$ to independent matrices U_0 and U_1 allows us to conclude that, under the null hypothesis $H_0 : \boldsymbol{\mu}_Y = 0$, and for large n, S_n approximatively follows an Hotelling T^2 distribution with q degrees of freedom; see Anderson (1984, p. 163). Therefore,

$$F \equiv \frac{(2J - q + 1)}{2J}\frac{S_n}{q}$$

follows a $F_{q,2J-q+1}$ distribution . Since the parameter J in the expression (3.15) is generally relatively large with respect to the dimension q of the process Y, the distribution of the statistic $\frac{1}{q}S_n$ can be approximated by an $F_{q,2J}$ distribution.

3.2.1 Test for non-correlation at individual lags

By Proposition 3.1, the transformed process Y_k defined by (3.12) also satisfies Assumption A.2, and the results of the previous section apply for Y_k. It allows us to propose the following procedure for H_0 against $H_{1k} : \Gamma_\mathrm{X}^{(12)}(k) \neq 0$.

Given the two realizations $\mathrm{X}^{(1)}(1), \ldots, \mathrm{X}^{(1)}(n)$ and $\mathrm{X}^{(2)}(1), \ldots, \mathrm{X}^{(2)}(n)$:

Step 1 Compute the transformed realization of length $n - |k|$:

$$\mathrm{Y}_k(t) = \begin{cases} \mathrm{X}^{(1)}(t) \otimes \mathrm{X}^{(2)}(t-k) \quad , \quad t = k+1, \ldots, n & \text{if} \quad k \geq 0, \\ \mathrm{X}^{(1)}(t) \otimes \mathrm{X}^{(2)}(t-k) \quad , \quad t = 1, \ldots, n+k & \text{if} \quad k < 0. \end{cases}$$

Step 2 With the realization obtained at step 1, compute the values of the periodogram $\{\mathrm{I}_Y^{(n)}(2\pi j/n)\ ;\ j = 1, \ldots, J\}$ and deduce the estimator $\mathrm{f}_Y^{(n)}(0)$ defined by (3.15).

Step 3 Compute the quadratic form

$$\mathrm{QS}(k) = n\overline{\mathrm{Y}}_k^{'}\{\mathrm{f}_Y^{(n)}(0)\}^{-1}\overline{\mathrm{Y}}_k/(2\pi d_1 d_2), \tag{3.17}$$

where $\overline{\mathrm{Y}}_k$ is the sample mean of the series $\mathrm{Y}_k(t)$.

Step 4 Reject H_0 at level α if $\mathrm{QS}(k) > F_{d_1d_2,2J-d_1d_2+1}(1-\alpha)$ where $F_{d_1d_2,2J-d_1d_2+1}(\alpha)$ is the αth quantile of the distribution $F_{d_1d_2,2J-d_1d_2+1}$.

3.2.2 Global test of non-correlation

The approach of the previous section can be extended to take into account an arbitrary finite number of lag cross-covariances and therefore, to construct a global test for H_0 against $H_1^{(M)}$. Let $k_1, \ldots, k_m$ be distinct lags, and let

$$\mathrm{Z}_m(t) = (\mathrm{Y}_{k_1}(t)', \ldots, \mathrm{Y}_{k_m}(t)')' \quad : \quad t \in \mathbb{Z}, \tag{3.18}$$

where each vector $\mathrm{Y}_{k_j}(t)$ is defined by (3.12). Then, the multivariate process $\{\mathrm{Z}_m(t)\}$ is of dimension $m \times d_1 d_2$ and each of its vector components $\{\mathrm{Y}_{k_j}(t)\}$ satisfies Assumption A.2 according to Proposition 3.1. Its mean $\boldsymbol{\mu}_Z(m)$ is given by

$$\mathrm{E}\{\mathrm{Z}_m(t)\} \equiv \boldsymbol{\mu}_Z(m) = (\boldsymbol{\mu}_Y^{'}(k_1), \ldots, \boldsymbol{\mu}_Y^{'}(k_m))^{'},$$

where $\boldsymbol{\mu}_Y(k_j)$ is defined by (3.13). The following general result, as Proposition 3.1, can be deduced from the multivariate version of Theorem 2.9.1 in Brillinger (1975).

Proposition 3.3 *If the process* X *satisfies Assumption A.2, then the process* Z_m *defined by (3.18) also satisfies Assumption A.2.*

Consider the realization $\mathrm{Z}_m(1), \ldots, \mathrm{Z}_m(n)$ of the process $\{\mathrm{Z}_m(t)\}$. For example, that realization can be obtained from a realization of length $n + K$ of the series $\{\mathrm{X}(t)\}$ where $K = \max\{|k_1|, \ldots, |k_m|\}$. Letting $\bar{\mathrm{Z}}_m$ and $\mathrm{f}_Z^{(n)}(\cdot)$ be the sample mean and the smooth periodogram estimator of the spectral density of the process Z_m, we can define the following statistic:

$$\mathrm{QS}_m = \frac{n}{2\pi}(\bar{\mathrm{Z}}_m - \boldsymbol{\mu}_Z(m))^{'}\{\mathrm{f}_Z^{(n)}(0)\}^{-1}(\bar{\mathrm{Z}}_m - \boldsymbol{\mu}_Z(m)) \tag{3.19}$$

>From Propositions 3.2 and 3.3, we can conclude that QS_m asymptotically follows an Hotelling T^2 distribution with md_1d_2 degrees of freedom.

Choosing $\{k_1, k_2, \ldots, k_m\} = \{-M, -M+1, \ldots, M-1, M\}$, the global statistic for testing H_0 against $H_1^{(M)}$ is the following.

Step 1 For $|k| \leq M$, compute

$$\mathrm{Y}_k(t) = \begin{cases} \mathrm{X}^{(1)}(t) \otimes \mathrm{X}^{(2)}(t-k) & , \quad t = k+1, \ldots, n \quad \text{if} \quad k \geq 0, \\ \mathrm{X}^{(1)}(t) \otimes \mathrm{X}^{(2)}(t-k) & ; \quad t = 1, \ldots, n+k \quad \text{if} \quad k < 0. \end{cases}$$

Step 2 Construct the vector of length $(2M+1)d_1d_2$

$$\mathrm{Z}_M(t) = (\mathrm{Y}_{-M}(t)', \ldots, \mathrm{Y}_0(t)', \ldots, \mathrm{Y}_{M-1}(t)', \mathrm{Y}_M(t)')' \ , \ t = M+1, \ldots, n-M.$$

Step 3 With the realization of length $n-2M$ of the process Z_M obtained at step 2, compute the estimator $\mathrm{f}_Z^{(n)}(0)$ and the quadratic form QS_M defined by (3.19), replacing n by $n-2M$.

Step 4 Reject H_0 at level α if

$$\frac{(2J - (2M+1)d_1d_2 + 1)}{2J(2M+1)d_1d_2} \mathrm{QS}_M > F_{(2M+1)d_1d_2, 2J-(2M+1)d_1d_2+1}(\alpha). \tag{3.20}$$

If the means of the subprocesses $\mathrm{X}^{(1)}$ and $\mathrm{X}^{(2)}$ are different from 0, then the process Y_k in (3.12) is replaced by $\mathrm{Y}_k(t) = (\mathrm{X}^{(1)}(t) - \overline{\mathrm{X}}^{(1)}) \otimes (\mathrm{X}^{(2)}(t-k) - \overline{\mathrm{X}}^{(2)})$, $t \in \mathbb{Z}$. As in the univariate case, the correction for the sample means does not affect the asymptotic distribution of the proposed statistics under Assumption A.2; see Brillinger (1978).

4 Test for non-correlation at positive or negative lags

In the analysis of economic time series, a question often raised is whether a set of variables causes another one in the sense of Granger (1969). The approach described in Section 3.1 can be easily adapted to test a sufficient condition for Granger causality. Let us consider the second-order stationary process $\mathrm{X} = \{\mathrm{X}(t) = (\mathrm{X}^{(1)}(t)', \mathrm{X}^{(2)}(t)') : t \in \mathbb{Z}\} = (\mathrm{X}^{(1)}, \mathrm{X}^{(2)})$ and without loss of generality, we can assume that its mean is 0. Further, let $\mathcal{I}_\mathrm{X}(t)$ be the Hilbert space generated by the components of $\mathrm{X}(s)$ for $s \leq t$, where the associated scalar product is the covariance operator. Let $\mathcal{I}_\mathrm{X}^{(h)}(t)$, $h = 1, 2$ be the closed subspace of $\mathcal{I}_\mathrm{X}(t)$, generated by the components of $\mathrm{X}^{(h)}(s)$ for $s \leq t$. The sets $\mathcal{I}_\mathrm{X}(t)$ and $\mathcal{I}_\mathrm{X}^{(j)}(t)$ are often called "information sets" in the econometric literature. For any information subset $\mathcal{I}(t-1)$ of $\mathcal{I}_\mathrm{X}(t-1)$, let $\mathcal{P}(X_l^{(h)}(t)/\mathcal{I}(t-1))$ denote the affine projection of $X_l^{(h)}(t)$ on $\mathcal{I}(t-1)$ (that is, the best unbiased linear prediction of the component $X_l^{(h)}(t)$ based on the variables in $\mathcal{I}(t-1)$ and the constant variable) and $\mathcal{P}(\mathrm{X}^{(h)}(t)/\mathcal{I}(t-1)) = (\mathcal{P}(X_1^{(h)}(t)/\mathcal{I}(t-1)), \ldots, \mathcal{P}(X_{d_h}^{(h)}(t)/\mathcal{I}(t-1))'$ the vector of predictors of $\mathrm{X}^{(h)}(t)$. Non causality in the sense of Granger (1969) is defined as follows (see also Boudjellaba, Dufour and Roy, 1992).

Definition 4.1 *The process $\mathrm{X}^{(i)}$ does not cause the process $\mathrm{X}^{(j)}$ if*

$$\mathcal{P}(\mathrm{X}^{(j)}(t)/\mathcal{I}_\mathrm{X}(t-1)) = \mathcal{P}(\mathrm{X}^{(j)}(t)/\mathcal{I}_\mathrm{X}^{(j)}(t-1))$$

for all t, where the equality holds in the L_2 sense, $i, j = 1, 2$ and $i \neq j$.

Then, the following result follows easily.

Proposition 4.1 *Let $\mathrm{X} = (\mathrm{X}^{(1)}, \mathrm{X}^{(2)})$ be a second order stationary process with mean 0. Then for $i, j = 1, 2, \ i \neq j$, we have that*

$$\Gamma_\mathrm{X}^{(ij)}(k) = 0 \ , \ k \geq 1 \quad \Longrightarrow \quad \mathrm{X}^{(j)} \quad \textit{does not cause} \quad \mathrm{X}^{(i)}.$$

Proof : We have $\Gamma_\mathrm{X}^{(ij)}(k) = \mathrm{E}[\mathrm{X}^{(i)}(t)\mathrm{X}^{(j)}(t-k)'] = 0$ for $k > 0$ implies that $\mathrm{X}^{(i)}(t)$ is orthogonal to the subspace $\mathcal{I}_\mathrm{X}^{(j)}(t-1)$ generated by the vectors $\mathrm{X}^{(j)}(t-k)$; $k \geq 1$. In terms of orthogonal projections, it means that $\mathcal{P}(\mathrm{X}^{(i)}(t)/\mathcal{I}_\mathrm{X}(t-1)) = \mathcal{P}(\mathrm{X}^{(i)}(t)/\mathcal{I}_\mathrm{X}^{(i)}(t-1))$, and therefore that $\mathrm{X}^{(j)}$ does not cause $\mathrm{X}^{(i)}$.

That sufficient condition can be useful to detect causality directions between $\mathrm{X}^{(1)}$ and $\mathrm{X}^{(2)}$. In that perspective, consider the two hypotheses of non-correlation at positive or negative lags defined respectively by

$$H_0^{(+)} \ : \ \boldsymbol{\rho}_\mathrm{X}^{(12)}(k) = 0, \ \text{for all } k > 0,$$

and

$$H_0^{(-)} \ : \ \boldsymbol{\rho}_{\rm X}^{(12)}(k) = 0, \text{ for all } k < 0.$$

Using relation (5) of Roy (1989), we can verify that under the hypothesis $H_0^{(+)}$, the asymptotic covariance structure of any finite set of sample cross-correlations at positive lags is identical to the one obtained under the hypothesis H_0 of non-correlation. In that case, the asymptotic covariance $\Sigma_{k,h}$ between the vectors $\text{vecR}_{\rm X}^{(12)}(k)$ and $\text{vecR}_{\rm X}^{(12)}(h)$ is given, for all $k, h > 0$, by:

$$\Sigma_{k,h} = \begin{cases} \Sigma_{h-k} & \text{if} \quad h-k \geq 0, \\ \Sigma'_{h-k} & \text{elsewhere}, \end{cases} \tag{4.1}$$

where Σ_l (with $l = h - k$) is defined by (2.8).

It is also easily verified that under $H_0^{(-)}$, the asymptotic covariance between $\text{vecR}_{\rm X}^{(12)}(k)$ and $\text{vecR}_{\rm X}^{(12)}(h)$ is also given by (4.1) for $k, h < 0$.

To test the hypothesis of non-correlation at positive lags $H_0^{(+)}$ against the alternative

$$H_1^{(+M)} \ : \ \boldsymbol{\rho}_{\rm X}^{(12)}(k) \neq 0, \text{ for at least one } k \text{ such that } 1 \leq k \leq M,$$

it is natural to base the test statistic on the cross-correlation $\text{R}_{\rm X}^{(12)}(k)$, for $1 \leq k \leq M$. With such a test, if $H_0^{(+)}$ is not rejected, we can conclude that $\text{X}^{(2)}$ does not cause $\text{X}^{(1)}$. However, when $H_0^{(+)}$ is rejected, nothing can be concluded about the causality of $\text{X}^{(2)}$ towards $\text{X}^{(1)}$.

As in Section 3.1, we can carry out simultaneous tests at lags $k = 1, 2, \ldots, M$ based on QR(k) or QS(k), defined respectively by (3.4) and (3.17), using the marginal level α/M. That approach based on Bonferroni inequality leads to a global conservative test.

A portmanteau type test that takes into account the complete asymptotic covariance structure is based on the vector $\text{r}_{\text{X}(+)}^{(12)} = (\text{vecR}_{\rm X}^{(12)}(1)', \ldots, \text{vecR}_{\rm X}^{(12)}(M)')'$. Let $\hat{\Sigma}^{(+)} = (\hat{\Sigma}_{k-h})_{1 \leq k,h \leq M}$ be the estimated asymptotic covariance matrix of $\text{r}_{\text{X}(+)}^{(12)}$ given by (3.3). The test statistic defined by

$$\text{QR}_M^+ = n \text{r}_{\text{X}(+)}^{(12)'} \hat{\Sigma}^{(+)^{-1}} \text{r}_{\text{X}(+)}^{(12)} \tag{4.2}$$

is asymptotically distributed as a chi square variable with $d_1 d_2 M$ degrees of freedom and an asymptotic test for $\text{H}_0^{(+)}$ can be deduced in the usual way. A test for $H_0^{(-)}$ can be derived in a similar manner using the vector $\text{r}_{\text{X}(-)}^{(12)} = (\text{vecR}_{\rm X}^{(12)}(-1)', \ldots, \text{vecR}_{\rm X}^{(12)}(-M)')'$ and the statistic

$$\text{QR}_M^- = n \text{r}_{\text{X}(-)}^{(12)'} \hat{\Sigma}^{(-)^{-1}} \text{r}_{\text{X}(-)}^{(12)}. \tag{4.3}$$

It is interesting to remark that the asymptotic covariance matrices $\Sigma^{(+)}$ and $\Sigma^{(-)}$ of the vectors $\text{r}_{\text{X}(+)}^{(12)}$ and $\text{r}_{\text{X}(-)}^{(12)}$ are similar, that is $\Sigma^{(+)} = \text{P}'\Sigma^{(-)}\text{P}$ for a certain orthogonal matrix $\mathbf{P}$.

5 Simulation results

5.1 Description of the experiment

In this section, we present the results of a small simulation experiment realized to compare the exact distributions of the statistics QR(k), QS(k), QR_M, and QS_M, with their respective asymptotic distributions under the null hypothesis H_0 of non-correlation. We also compare the empirical power of these tests under a particular alternative to the empirical power of the multivariate version of Haugh's test described in El Himdi and Roy (1997). With this aim, we analyzed the empirical frequencies of rejection of the null hypothesis by tests with three different nominal levels (1, 5 and 10 percent) for each of two series lengths ($n = 100$ and 200) anf for two global ARMA models for $\text{X}^{(1)}$ and $\text{X}^{(2)}$. The two models are described in Table 1. In Model A, $\text{X}^{(1)}$ and $\text{X}^{(2)}$ are uncorrelated and allows us to evaluate the exact levels of the proposed tests. In Model B, $\text{X}^{(1)}$ and $\text{X}^{(2)}$ are weakly correlated through their corresponding innovation processes that are only correlated at lag 0. For each model, 10 000 independent realizations were generated.

Model	Equation	Ω_a
A	$\begin{bmatrix} \text{X}^{(1)}(t) \\ \text{X}^{(2)}(t) \end{bmatrix} = \begin{bmatrix} \Phi_1 & 0 \\ 0 & \Phi_2 \end{bmatrix} \begin{bmatrix} \text{X}^{(1)}(t-1) \\ \text{X}^{(2)}(t-1) \end{bmatrix} + \begin{bmatrix} \text{a}^{(1)}(t) \\ \text{a}^{(2)}(t) \end{bmatrix}$	$\begin{bmatrix} \Omega_1 & 0 \\ 0 & \Omega_2 \end{bmatrix}$
B	$\begin{bmatrix} \text{X}^{(1)}(t) \\ \text{X}^{(2)}(t) \end{bmatrix} = \begin{bmatrix} \Phi_1 & 0 \\ 0 & \Phi_2 \end{bmatrix} \begin{bmatrix} \text{X}^{(1)}(t-1) \\ \text{X}^{(2)}(t-1) \end{bmatrix} + \begin{bmatrix} \text{a}^{(1)}(t) \\ \text{a}^{(2)}(t) \end{bmatrix}$	$\begin{bmatrix} \Omega_1 & \Omega_{12} \\ \Omega_{12} & \Omega_2 \end{bmatrix}$

Parameters

$$\Phi_1 = \begin{bmatrix} 0.4 & 0.1 \\ -1.0 & 0.5 \end{bmatrix} \quad \Phi_2 = \begin{bmatrix} -1.5 & 1.2 \\ -0.9 & 0.5 \end{bmatrix} \quad \Omega_1 = \begin{bmatrix} 1.0 & 0.5 \\ 0.5 & 1.07 \end{bmatrix}$$

$$\Omega_2 = \begin{bmatrix} 1.0 & 0.75 \\ 0.75 & 1.0 \end{bmatrix} \quad \Omega_{12} = \begin{bmatrix} 0.15 & 0.0 \\ 0.0 & 0.15 \end{bmatrix}$$

Table 1: Models used in the simulation study.

For each model, the experiment proceeded in the following way.

1. 4-variate independent $\mathrm{N}(0, \Omega_{\mathrm{a}})$ innovations were generated using the NAG subroutine G05EAF.

2. The values $\mathrm{X}(1), \ldots, \mathrm{X}(N)$ were obtained by solving the difference equation defining the model. For each model, 10 000 independent realizations of length N=200 were generated; The value X(1) was generated from the exact distribution $\mathrm{N}(0, \Gamma_{\mathrm{X}}(0))$ of $\mathrm{X}(t)$ using an algorithm of Ansley (1980) and the other observations were obtained using $N-1$ values of $\mathrm{a}(t)$ that were also independent of X(1).

3. For each realization $\mathrm{X}(1), \ldots, \mathrm{X}(200)$, the test statistics were computed for the first n observations of the time series for $n = 100$ and 200. The values of the statistics $\mathrm{QR}(k)$, $\mathrm{QR}^*(k)$ and $\mathrm{QS}(k)$ were computed for $k = -12, \ldots, 12$, and those for QR_M and QS_M for $M = 1, 2, \ldots, 12$ (when it was possible). For each test, the value of the statistic was compared with the critical value obtained from the corresponding chi square distribution. To compute the statistics QR ($\mathrm{QR}(k)$, $\mathrm{QR}^*(k)$ and QR_M), we used the modified Bartlett window as the weight function and the truncation points $T_n = H\sqrt{n}$, $H = 1, 3, 5$. However, only the results for $H = 3$ are reported. Similarly, in the calculation of the QS statistics ($\mathrm{QS}(k)$ and QS_M), we have to evaluate the smoothed periodogram (3.15) at the frequency 0. The values for J were obtained from a preliminary simulation. With the $\mathrm{QS}(k)$ statistics, we used $J = 10$ for $n = 100$ and $J = 20$ for $n = 200$. With QS_M, we employed $J = 30$ for $n = 100$ and $J = 60$ for $n = 200$. In order that the second degree of freedom $2J - (2M+1)d_1 d_2 + 1$ of the F distribution in (3.20) be positive, we must have $M \leq 6$ when $n = 100$ and $M \leq 14$ when $n = 200$.

5.2 Behavior of the tests at individual lags

The empirical levels of tests at individual lags based on $\mathrm{QR}(k)$, $\mathrm{QR}^*(k)$ and $\mathrm{QS}(k)$ are reported in Table 2, for $|k| = 0, 1, 2, 3, 4, 5, 6, 8, 10$ at $\alpha = 0.05$. Due to space constraints, the results for $\alpha = 0.01$ and $\alpha = 0.10$ are not presented. With 10 000 realizations, the standard error of these empirical levels is 0.22%. Firstly, for $\mathrm{QR}(k)$, the chi square distribution provides a relatively poor approximation for the larger lags, specially for series of 100 observations. The approximation for the modified statistic $\mathrm{QR}^*(k)$ is much better. In most cases, the empirical levels are within 3 standard errors of 5% for $n = 100$ or 200. The rejection rates with the statistics $\mathrm{QS}(k)$ are slightly above 5% with $n = 100$ and are all within 2 standard errors of 5% when $n = 200$.

$k =$	-10	-8	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	8	10
n = 100																	
$QR(k)$	2.7	3.1	3.5	3.5	3.4	3.8	4.0	4.4	4.3	4.2	4.3	3.9	3.8	3.7	3.3	3.0	2.6
$QR^*(k)$	4.6	4.4	4.5	4.3	3.8	4.3	4.5	4.5	4.3	4.4	4.7	4.4	4.4	4.6	4.5	4.3	4.4
$QS(k)$	6.3	6.1	6.5	6.5	6.1	6.1	6.1	6.3	6.0	6.5	6.3	6.0	6.6	6.3	6.4	6.0	5.8
n = 200																	
$QR(k)$	3.7	4.0	3.8	4.1	4.4	4.3	4.5	4.4	4.5	4.5	5.0	4.3	4.3	4.5	4.1	4.2	3.7
$QR^*(k)$	4.8	4.6	4.2	4.6	4.7	4.7	4.7	4.5	4.5	4.6	5.1	4.4	4.7	5.0	4.7	4.9	4.9
$QS(k)$	5.0	5.0	5.3	5.3	5.3	5.0	5.3	5.2	4.8	5.0	5.1	5.5	5.4	5.3	4.8	5.3	5.2

Table 2: Empirical levels (in percentage) of tests at individual lags based on $\mathrm{QR}(k)$, $\mathrm{QR}^*(k)$ and $\mathrm{QS}(k)$ with model A, at the nominal level $\alpha = 0.05$.

The rejection rates at the level $\alpha = 0.05$ of the null hypothesis of non-correlation, when the two series $\mathrm{X}^{(1)}$ and $\mathrm{X}^{(2)}$ are generated under the alternative defined by model B, are given in the Table 3. We only present the empirical power for $\mathrm{QR}^*(k)$ and $\mathrm{QS}(k)$, whose levels are reasonably well controlled.

For a given k, the power of $\mathrm{QR}^*(k)$ and $\mathrm{QS}(k)$ are quite similar. As a function of k, the powers of both tests rapidly decrease and the highest values are obtained at the small lags 0, ± 1, ± 2.

$k=$	-10	-8	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	8	10
n = 100																	
$QR^*(k)$	4.2	4.2	4.3	4.4	4.0	5.2	11.4	25.1	33.4	25.3	16.1	8.1	5.2	4.2	4.4	4.1	4.2
$QS(k)$	5.9	6.2	6.7	6.3	7.2	9.6	15.4	22.5	31.6	23.0	12.7	7.2	6.3	6.2	6.6	6.0	6.2
n = 200																	
$QR^*(k)$	4.8	4.7	4.4	4.6	4.8	7.8	22.5	53.4	67.9	52.5	33.8	14.1	7.1	5.1	4.4	4.8	4.4
$QS(k)$	5.0	5.3	5.2	5.4	7.3	13.7	29.6	45.3	63.8	47.4	21.1	8.6	5.8	5.5	5.0	5.4	5.5

Table 3: Empirical powers (in percentage) of tests at individual lags based on $\mathrm{QR}^*(k)$ and $\mathrm{QS}(k)$ with model B, at the nominal level $\alpha = 0.05$.

$M=$	1	2	3	4	5	6	7	8	9	10	11	12
n = 100												
QH_M^*	4.6	4.5	4.7	4.5	4.6	4.7	4.7	5.0	4.9	4.9	5.0	5.0
QR_M	1.4	0.4	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
QS_M	2.9	3.6	4.0	3.9	3.4	3.7						
n = 200												
QH_M^*	5.1	4.8	4.7	4.8	4.8	4.8	4.7	4.9	4.9	5.0	4.9	5.0
QR_M	2.0	1.2	0.8	0.5	0.2	0.1	0.1	0.0	0.0	0.0	0.0	0.0
QS_M	2.9	4.0	4.3	4.3	4.3	4.5	4.4	4.4	4.2	4.1	4.3	4.1

Table 4: Empirical levels (in percentage) of the tests QH_M^* (El Himdi and Roy, 1997), QR_M and QS_M with model A, at $\alpha = 0.05$.

$M=$	1	2	3	4	5	6	7	8	9	10	11	12
n = 100												
QH_M^*	82.6	70.3	60.5	54.1	48.2	43.5	40.3	37.6	35.3	33.4	31.2	30.0
QR_M	76.6	64.3	55.7	47.0	39.5	34.2	28.7	26.7	23.3	20.8	19.3	16.7
QS_M	18.7	12.9	9.7	8.5	8.1	6.6						
n = 200												
QH_M^*	99.7	98.8	96.9	94.6	92.0	88.8	85.7	82.9	80.6	77.6	75.1	72.5
QR_M	99.6	98.1	96.2	93.0	88.8	85.0	79.9	74.8	71.1	67.8	63.3	60.0
QS_M	48.0	35.2	26.8	22.0	18.4	15.8	14.0	11.8	10.8	9.8	8.8	8.3

Table 5: Empirical powers (in percentage) of the global tests QH_M^*, QR_M and QS_M with model B, at $\alpha = 0.05$. For QR_M and QS_M, the exact critical values obtained in the level study were used.

5.3 Behavior of the global tests

As a benchmark for the power analysis of the proposed nonparametric tests, we used the multivariate version of Haugh's test studied by El Himdi and Roy (1997). It is a parametric test based on the residual cross-correlation matrices $\mathrm{R}_{\hat{\mathrm{a}}}^{(12)}(k)$, $|k| \leq M$, between the two residual series $\hat{\mathrm{a}}^{(1)}(t)$ and $\hat{\mathrm{a}}^{(2)}(t)$ resulting from fitting multivariate ARMA models to the original series $\mathrm{X}^{(1)}(t)$ and $\mathrm{X}^{(2)}(t)$. Here we estimated the true AR(1) models for each series and the resulting statistic is noted QH_M^* as in El Himdi and Roy (1997).

The empirical levels of the tests QH_M^*, QR_M and QS_M are presented in Table 3. As already observed by El Himdi and Roy (1997), the exact level of QH_M^* is quite close to the nominal level. However, the tests based on QR_M are very conservative. The exact distribution of QR_M is considerably shifted to the left of the corresponding chi square asymptotic distribution. Various modifications were tried: 1) the modification $n/(n-|k|)$ applied at each lag k; 2) a translation of the whole distribution based on M and n as suggested in Li and McLeod (1981). No satisfactory modification has yet been obtained. The test QS_M is slightly conservative but except at $M = 1$, its exact level is at least 4% with $n = 200$.

For various values of M, the empirical powers of QH_M^*, QR_M and QS_M are given in Table 5. The power of QH_M^* is based on its asymptotic critical value whilst those of QR_M and QS_M were obtained from the exact critical values obtained in the level study. We make the following observations. First, QR_M seems considerably more powerful than QS_M even at $n = 200$. The power of both tests decreases as M increases as it is usually the case with portmanteau type tests. Second, the power of QR_M is slightly smaller than the one of QH_M^*. At least for the particular model considered, the loss of power resulting from the use of the nonparametric test (with the exact critical values) rather than the parametric test QH_M^* is quite reasonable.

6 Application

In this example, we consider a set of seven American and Canadian quarterly economic indicators used in a study of the Canadian monetary policy; for a description of the data, see Racette and Raynauld (1992). The goal of this section is to illustrate the use of the nonparametric tests introduced in the previous sections to explore the relationships between the Canadian and American economies. The Canadian economic indicators are the gross domestic product (GDP) in constant dollars of 1982, the implicit price index of the gross domestic production (GDPI), the nominal short-term interest rate (TX.CA) and the monetary basis value (M1). The other three variables represent the American gross national product in constant dollars of 1982 (GNP), the implicit price index of the American gross national product (GNPD) and the nominal short-term American interest rate (TX.US). In this study, the observation period is from the first quarter of 1970 through the last quarter of 1989, giving 80 observations for each series. The logarithmic transformation was applied to the M1 time series to stabilize the variance and all series but TX.CA were differenced, to have stationarity.

In the following, the two Canadian and American vector time series, denoted $\{\mathrm{X}^{(1)}(t)\}$ and $\{\mathrm{X}^{(2)}(t)\}$ respectively, are defined by:

$$\mathrm{X}^{(1)}(t) = \begin{bmatrix} \frac{1}{1000}(1-B)GDP(t) \\ 10(1-B)GDPI(t) \\ TX.CA(t) \\ 100(1-B)\ln(M1(t)) \end{bmatrix}, \quad \mathrm{X}^{(2)}(t) = \begin{bmatrix} \frac{1}{10}(1-B)GNP(t) \\ 10(1-B)GNPD(t) \\ TX.US(t) \end{bmatrix}.$$

The multiplicative factors in the definition of these series were chosen in order to obtain variances of the same order of magnitude within each series.

To test the hypothesis of non-correlation between the two economic time series, we employed the tests $\mathrm{QR}^*(k)$ at individual lags. The asymptotic covariance structure of the serial correlations was estimated with the modified Bartlett window and the truncation point $T_n = H\sqrt{n}$, $H = 3$. The values of $\mathrm{QR}^*(k)$, $|k| = 0, 1, 2, \ldots, 12$ are displayed in Figure 1. To test the null hypothesis H_0 of non-correlation at the significance level $\alpha = 0.05$ the critical value is the quantile $\chi^2_{12,0.95} = 21.02$ and we reject H_0 if $\mathrm{QR}^*(k)$ is greater than that value. We see from Figure 1 that H_0 is rejected at the 6 lags $k = -1, 0, 1, 2, 3, 4$. With the simultaneous tests $\mathrm{QR}^*(k)$, $|k| \le M$, at the marginal level $\alpha_M = \alpha/(2M+1)$, the global level is at most α by Bonferroni inequality. Again, the null hypothesis H_0 is clearly rejected with $M = 4, 8$ and 12. With the global test QR_M, we reject at the 5% level with $M = 1$ but we do not reject with $M = 2, 3, \ldots, 12$. There are two possible reasons for that. First, the true level is possibly much less than 5% (see Table 4) and even with a well controlled level, the power of QR_M decreases as M increases (see Table 5). The simultaneous tests $\mathrm{QR}^*(k)$, $|k| \le M$, are more convincing than QR_M in this example.

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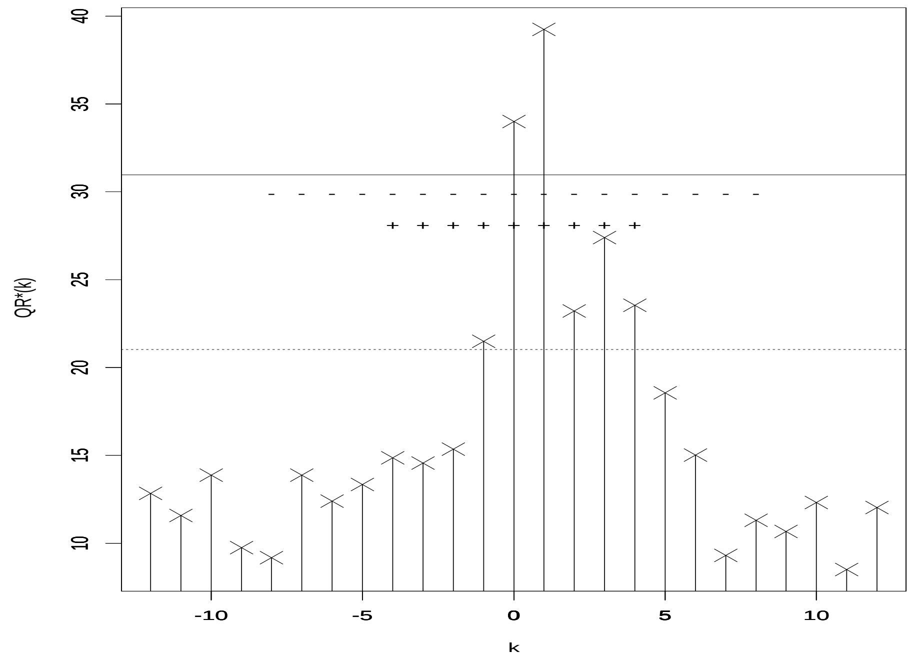

Figure 1: Values of the statistic $\text{QR}^*(k)$, $|k| = 0, 1, 2, \ldots, 12$. The dotted line represents the marginal critical value at the significance level $\alpha = 0.05$. The other lines represent the critical values, at the global level $\alpha = 0.05$, for the simultaneous tests at lags $|k| = 0, 1, 2, \ldots, M$, with $M = 4(+)$, $8(-)$ and 12 (solid line).

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