Problem 6

Solar Energy Portfolio Analysis

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Summary of the problem

We want to form a portfolio of solar plants to optimize investments in that type of energy.

Some governments (e.g. Ontario) offer a fixed price for solar power. This price is fixed and guaranteed for 20 years. All produced power will be bought.

The output of a system is in direct relation with the irradiation of its location.

Irradiation is random. Data are available to estimate its distribution:

- Environment Canada provides hourly measures of irradiation over some locations (including airports) for the period 1953 – 2003. The number of missing values is reasonably low.
- Turquoise has hourly irradiation data for any location in Canada that is determined from satellite images (covering 11 years).

The only random element is the level of radiation.

Notation

We are considering m different sites over 20 years. We want to invest C \$ for the development of some of these sites.

We need to decide the size s_i of the plant at location i. The cost of developing site i is

$$c_i = v_i s_i$$

Fixed cost to open a new site are negligible compared to the cost of solar panels.

We suppose that we are under the special regime where a fixed price p \$/MWh is guaranteed by the government.

At time $t = 1, \ldots, 20$ (years) and location i, let

$$I_{it} = \text{Average daily irradiation for site } i \text{ during year } t \ (W/m^2)$$

the annual output of the plant is in direct relation with I_{it}. To simplify things, we will suppose that we can summarize the efficiency of the plant by a single multiplicative constant (a location specific efficiency factor), hence

$$X_{it} = \text{output of site } i \text{ for year } t \ (MWh)$$

Typically, 70% of the cost of such a project is borrowed. We suppose that the annual interest rate is u and that equal annual payments on the debt are required, namely

$$M_t = 0.7(\boldsymbol{s}^T\boldsymbol{v})\left\{\frac{u}{1-(1+u)^{-n}}\right\} I(t \leq n)$$

where $\boldsymbol{s}^T\boldsymbol{v}$ is the total cost of the project, n is the number of years to amortize the loan.

Let also suppose a fixed risk-free interest rate j. The accumulated profits will have that return. We thus have:

$$\boldsymbol{s}^T\boldsymbol{v} \leq C$$

$$\text{Profit year } t = P_t = -M_t + p\sum_{i=1}^{m} s_i X_{it} = -M_t + p\boldsymbol{s}^T\boldsymbol{X}_t$$

$$L_t = \sum_{k=1}^{t} (1+j)^{t-k} P_k$$

$$P(\text{default at year } t) = P(L_t < 0)$$

Portfolio

The usual portfolio problem consists in minimizing the risk with respect to a given annual return r and other constraints.

Objective function

There is a choice to be made on the measure of risk. Some options are:

$$var\left(\sum_{i=1}^{m} s_i X_{it}\right)$$

$$VaR_\alpha = q_\alpha(L_{20}) = \arg\max_x \{P(L_{20} \leq x) \geq \alpha\}$$

$$cVaR_\alpha = E\{L_{20} | L_{20} < q_\alpha(L_{20})\}$$

Constraints

$$C(1+r)^{20} \leq E(L_{20}) + \text{value of the sites in 20 years}$$

$$0 \leq s_i \leq S_i, i = 1, \dots, 20$$

$$P(L_t < 0) \leq \beta, t = 1, \dots, 20$$

Stochastic representation of X_{it}

The distribution of X_{it} will determine how we calculate all terms with $P()$, $E()$ or $var()$.

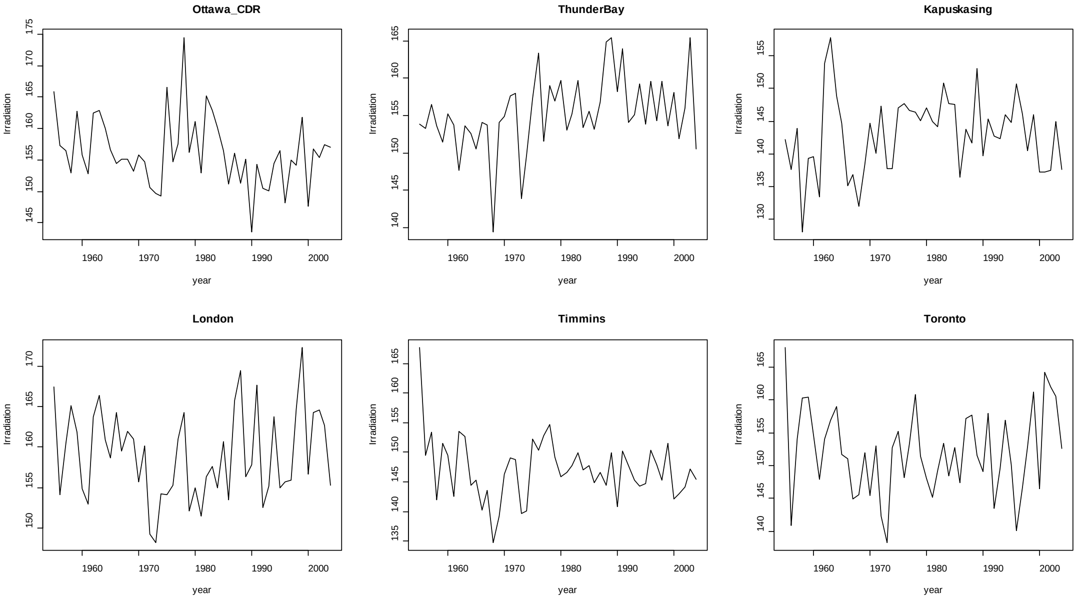
We have complete annual data for 50 years and 14 locations in Ontario.

The following plots show 6 randomly selected locations among the 14 (because plots would be too small and unreadable otherwise).

Questions:

- Is there temporal dependence?
  Easier if there is none.
- Is there geographical dependence?
  We expect so – this dependence is critical to the portfolio problem.
- What marginal distribution is suitable for the output at each site?
  Normal would be simplest.
- How should the dependence between sites be modelled?
  Multivariate normal would be simplest.
- How should temporal dependence be accounted for?
  If required.

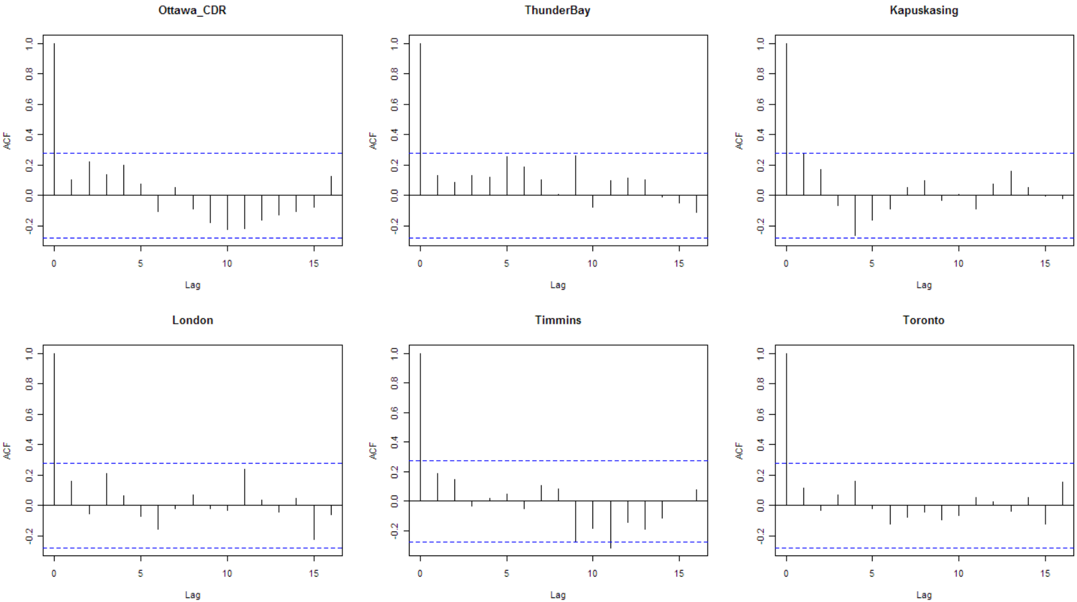
General shape of the time series



...shows no apparent trend or change of regime.

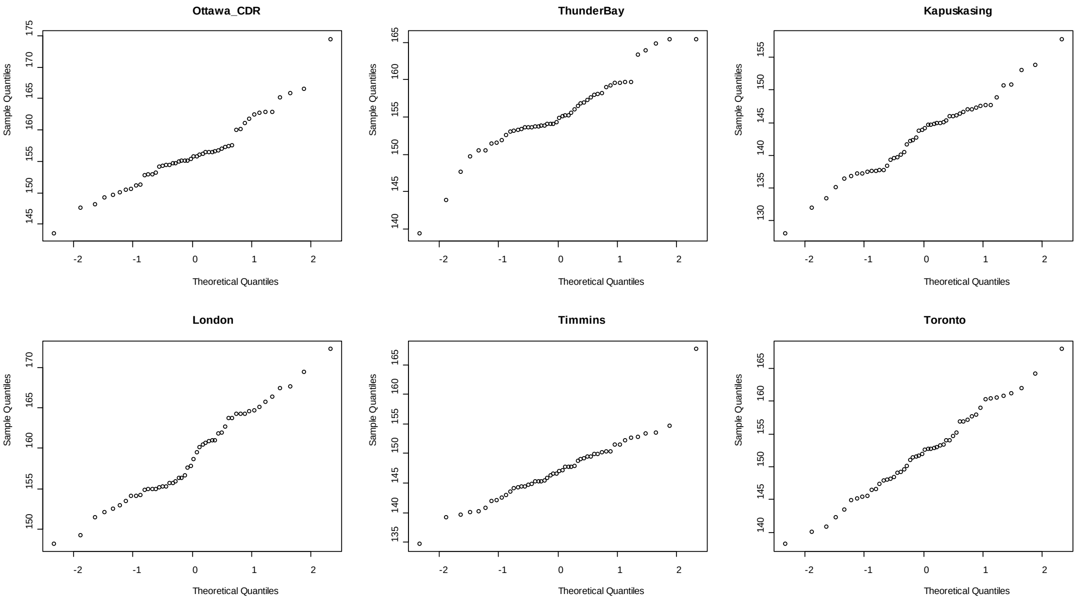
Temporal correlation

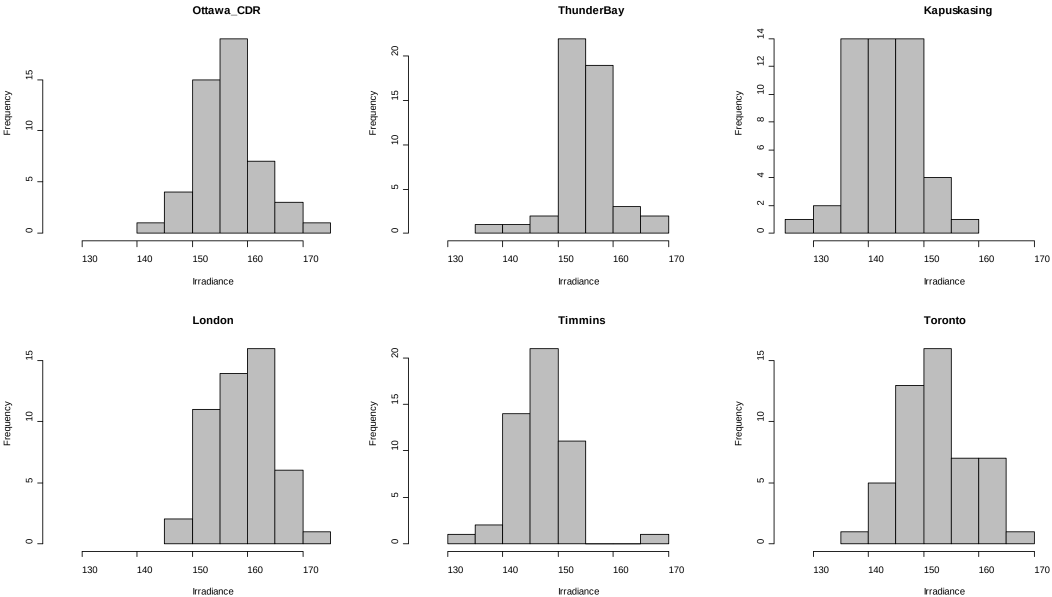
Temporal dependence would show on these correlograms as bars exceeding the dashed blue lines. It seems reasonable to assume there is no temporal dependence.



Marginal distributions

The following normal quantile plots look like a straight line if the data are normal, and depart from the line if normality is not reasonable.





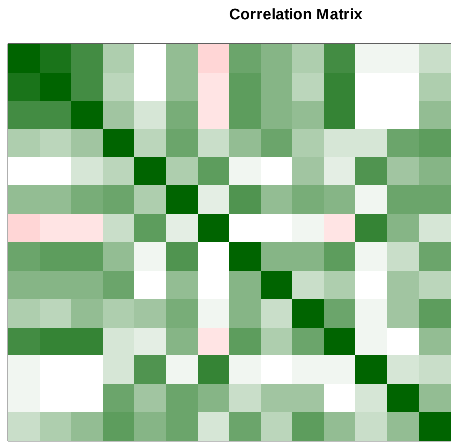
Except for one outlying value in Timmins, there is no strong sign of departure from a normal distribution. The fit is certainly not perfect, but it is far from unreasonable.

Geographical dependence

Since there is (surprisingly) no temporal dependence, we can see yearly irradiation as independent draws from a multivariate distribution.

The simplest solution is to suppose a multivariate normal distribution.

The MVN features no tail dependence. This means that the joint probability of extreme good or bad years may be underestimated. An analysis of the joint tails could call for a different (copula-based) model, but the optimization would then be much more difficult.



Instead of providing a table with numbers, the figure on the right shows the correlation matrix where dark green is 1, white is 0 and dark red is -1.

Note the small negative values in pink.

Dependence for the time component

Since there is no time dependence, we do not need to select a model for that component.

Conclusion (for the stochastic model)

The simplest model: Multivariate normal without temporal dependence seems to fit reasonably well. That will simplify our life in terms of calculations!

Implementation of a specific problem

We solve the program using reasonable values. We choose to

$$\min_{w.r.t.} cVaR_\alpha$$
$$C(1+r)^{20} \leq E(L_{20}) + V$$
$$0 \leq s_i \leq S_i, i = 1, \dots, 20$$
$$P(L_t < 0) \leq \beta, t = 1, \dots, 20$$

Simplification

The probability of default is decreasing with time (as a consequence of the lack of temporal dependence and the fact that having not defaulted yet means we have set money aside).

The constraint $P(P_1 < 0) \leq \beta$ is quadratic. We decided to leave it aside and check if the solution found respects it or not.

Under some conditions (that are respected in our case) including multivariate normality, Rockafellar & Uryasev (JofRisk, 2000) show that the minimization of $cVaR_\alpha$ is equivalent to minimizing the variance $\boldsymbol{s}^T \Sigma \boldsymbol{s}$.

Developing the expressions, we get:

$$\min_{w.r.t.} \boldsymbol{s}^T \Sigma \boldsymbol{s}$$
$$C(1+r)^{20} \leq \boldsymbol{s}^T\{\kappa(20)p\boldsymbol{\mu} - 0.7K\kappa(n)(1+j)^{20-n}\boldsymbol{v}\} + V$$
$$0 \leq s_i \leq S_i, i = 1, \dots, 20$$

Where $K = \left(\frac{u}{1-(1+u)^{-n}}\right)$, $\kappa(n) = \frac{(1+j)^n - 1}{j}$ and $V =$ value of the sites in 20 years.

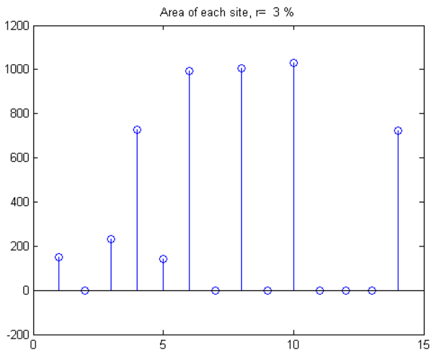
The probability of default can be expressed as

$$P(P_1 < 0) = \Phi\left\{\frac{-\boldsymbol{s}^T(p\boldsymbol{\mu} - 0.7K\boldsymbol{v})}{p(\boldsymbol{s}^T\Sigma\boldsymbol{s})^{1/2}}\right\}$$

- We determine $\boldsymbol{\mu}$ and Σ from the data set and a conversion factor.
- We set $u = 0.06, n = 7, j = 0.02$.
- $p = 820\ \$/MWh$
- $v_i = 650 \frac{\$}{m^2}$
- $C = 20\,000\,000$

If the investment is less than C, It is assumed that the rest is invested at the risk free interest rate. Its value is included in V above.

For different rates of return, we have a similar situation.



Note that the locations are respectively:

Earlton, Kapuskasing, Kenora, London, NorthBay, Ottawa_CDR, Ottawa_NRC, Sioux, Sudbury, ThunderBay, Timmins, Toronto, TorontoMetRes, TorontoPearson.

Legend: zero, small, large, largest.

The solutions above were found using Matlab, but they can also be calculated using the (more business-friendly) solver of Excel.

Extensions

- Adding annual maintenance cost does not change the problem.
- Adding a fixed cost (not per meter square) to the development of a site has major consequences on the difficulty of the problem.
- If the multivariate model does not fit, then the optimization becomes (much) harder as we need to calculate an integral to evaluate $sVaR$.
- Temporal dependence would make the resolution of this problem much trickier.

Notes about the literature

While there are many articles on energy portfolios, none of the papers we read included repayment of the debt in the formulation of the problem.