Estimating the plank production by Batch sampling

Second Montreal Industrial Problem Solving Workshop

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Introduction

Productivity issue

Chopping log = many kinds of planks

Input log volume $\longrightarrow$ Predictable output plank volume

It is difficult to predict the distribution among planks of varying qualities.

Estimate production distribution = Estimate logs cost

Plan

1. Problem Introduction
   - Problem Context
   - Problem's Data
   - Current problem
2. Solution Model
   - Problem modeling
   - Data and variables of the problem
   - Problem formulation
   - Problem formulation alternative
   - Problem formulation alternative 2
   - Problem formulation alternative 3
   - Problem formulation alternative 4
3. Application
   - Discussion

Problem Context

Source : Exact Modus

Simplified plank production chain.

Problem's Data

Each batch contains logs described by :

- length,
- diameter,
- colour,
- number of clear sides,
- volume (Board Foot Measure).

The batches are numbered.

Planks are described by :

- original batch ,
- length,
- thickness,
- width,
- quality,
- volume (Board Foot Measure).

Current Problem

One knows what is the original batch of each plank

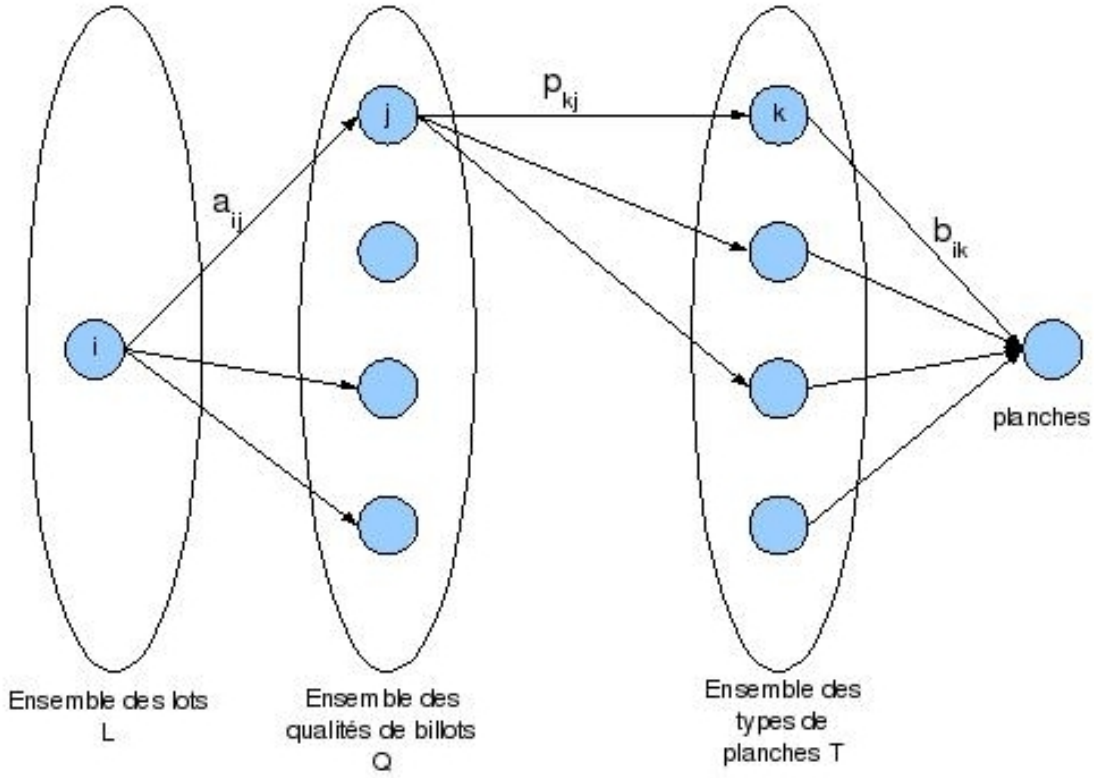
One doesn't know what is the original log of each plank

Goal :
We want to find a way to evaluate what kind of planks will be produced with a given log.

Data validation

- Large gap observed between total BFM of logs and total BFM of planks (20 % created) but " Rien ne se perd, rien ne se crée, tout se transforme " *Lavoisier*,
- Unstandardize batchs quality record.

Problem modeling



Data and variables of the problem

The data of the problem are :

- L : the set of batches,
- Q : the set of log types,
- T : the set of plank types,
- a_{ij} : the volume of j-type log in the batch i (in BFM)
- b_{ik} : the volume of k-type planks obtained with the batch i

The variables are :

- p_{kj} : proportion of k-type planks obtained with a log i,
- $\varepsilon_{ik}^{-}, \varepsilon_{ik}^{+}$: slack variables

Problem formulation

$$\min \sum_{i \in L, k \in T} (\varepsilon_{ik}^{+} + \varepsilon_{ik}^{-})$$

s.c. :

$$\sum_{j \in Q} a_{ij} p_{kj} + \varepsilon_{ik}^{-} - \varepsilon_{ik}^{+} = b_{ik} \quad , \quad \forall i \in L, \ \forall k \in T$$

$$\sum_{k \in T} p_{kj} = \alpha \quad , \quad \forall j \in Q$$

$$p_{kj} \geq 0 \quad , \quad \forall k \in T, \ \forall j \in Q$$

$$\varepsilon_{ik}^{+} \geq 0 \quad , \quad \varepsilon_{ik}^{-} \geq 0 \quad , \quad \forall i \in L, \ \forall k \in T$$

Problem formulation alternative

$$\min \sum_{i \in L, k \in T} \frac{1}{b_{ik}} (\varepsilon_{ik}^{+} + \varepsilon_{ik}^{-})$$

s.c. :

$$\sum_{j \in Q} a_{ij} p_{kj} + \varepsilon_{ik}^{-} - \varepsilon_{ik}^{+} = b_{ik} \quad , \quad \forall i \in L, \ \forall k \in T$$

$$\sum_{k \in T} p_{kj} = \alpha \quad , \quad \forall j \in Q$$

$$p_{kj} \geq 0 \quad , \quad \forall k \in T, \ \forall j \in Q$$

$$\varepsilon_{ik}^{+} \geq 0 \quad , \quad \varepsilon_{ik}^{-} \geq 0 \quad , \quad \forall i \in L, \ \forall k \in T$$

Problem formulation alternative 2

$$\min \sum_{i \in L, k \in T} w_{ik}(\varepsilon_{ik}^{+} + \varepsilon_{ik}^{-})$$

s.c. :

$$\sum_{j \in Q} a_{ij} p_{kj} + \varepsilon_{ik}^{-} - \varepsilon_{ik}^{+} = b_{ik} \quad , \quad \forall i \in L, \ \forall k \in T$$

$$\sum_{k \in T} p_{kj} = \alpha \quad , \quad \forall j \in Q$$

$$p_{kj} \geq 0 \quad , \quad \forall k \in T, \ \forall j \in Q$$

$$\varepsilon_{ik}^{+} \geq 0 \quad , \quad \varepsilon_{ik}^{-} \geq 0 \quad , \quad \forall i \in L, \ \forall k \in T$$

Problem formulation alternative 3

$$\min \quad \varepsilon$$

s.c. :

$$\sum_{j \in Q} a_{ij} p_{kj} + \varepsilon_{ik}^{-} - \varepsilon_{ik}^{+} = b_{ik} \quad , \quad \forall i \in L, \ \forall k \in T$$

$$\sum_{k \in T} p_{kj} = \alpha \quad , \quad \forall j \in Q$$

$$p_{kj} \geq 0 \quad , \quad \forall k \in T, \ \forall j \in Q$$

$$0 \leq \varepsilon_{ik}^{+} \leq \varepsilon, \ 0 \leq \varepsilon_{ik}^{-} \leq \varepsilon \qquad \forall i \in L \ \forall k \in T$$

Problem formulation alternative 4

$$\min \sum_{i \in L, k \in T} \varepsilon_{ik}^2$$

s.c. :

$$\sum_{j \in Q} a_{ij} p_{kj} + \varepsilon_{ik} = b_{ik} \quad , \quad \forall i \in L, \ \forall k \in T$$

$$\sum_{k \in T} p_{kj} = \alpha \quad , \quad \forall j \in Q$$

$$p_{kj} \geq 0 \quad , \quad \forall k \in T, \ \forall j \in Q$$

Discussion

Standardize batchs quality (for logs and planks).

Choose appropriate log types based on diameter AND number of clear sides.

More accurate formula for BFM prediction needed.

Validate the model on large data sets.