ADVANCED TUTORIAL

Monte Carlo and Quasi-Monte Carlo Methods in Finance

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In computational finance, MC/QMC simulation is one of the principal numerical tools. How much is a financial security worth? What risks are involved in holding a financial portfolio, how large are they, and what can be done to mitigate them? What portfolio best suits one's financial goals? What do security prices imply about stochastic models of financial markets? Simulations of financial markets are used in answering all of these questions: given a stochastic model of the markets, MC/QMC methods provide an approximation to a functional of a relevant distribution, such as the expectation or a quantile of the distribution of wealth at a future date. We give an overview of computational problems in derivative security pricing, risk management, portfolio optimization, and model calibration, and explain the ways in which simulation is applied to them.

One theme is computational efficiency of simulation procedures, emphasizing aspects of efficiency improvement specific to finance. For example, it is often possible to find very good control variates in financial simulations and to make Quasi-Monte Carlo work well even on very high-dimensional problems. Also, the simulations of financial markets used in computational finance typically take the form of exact or approximate solution of stochastic differential equations (SDEs) over a finite time horizon. We discuss the implications of an analysis of the error due to discretizing SDEs for simulation efficiency.

Another theme is ways in which simulation methodology has been cleverly extended to do more than approximate the distribution of a financial random variable. We present good simulation methods for sensitivity analysis: what is the rate of change of an expectation as underlying financial variables or model parameters change? We explain how simulation can be used in studying American-style options, which involve an optimal stopping problem. We consider the interplay between simulation and optimization when the goal is to find an optimal trading strategy or a best-fitting model. We also discuss the application of response surface modeling in financial simulations.