Prototype of a statistical / dynamical hybrid model for high-resolution hydrodynamic simulations

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Environnement et
Changement climatique Canada

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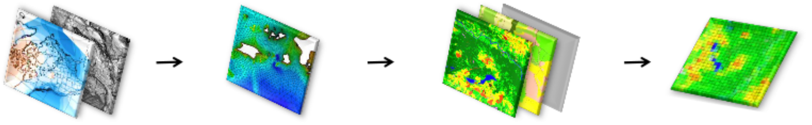
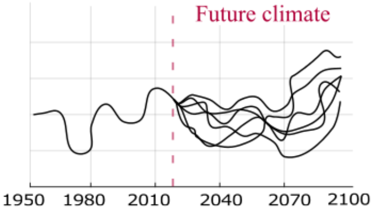
Modeling current and future water level fluctuations in the Great Lakes: temporal evolution of coastal wetlands under various climate scenarios

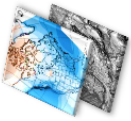


Climate model	Hydrodynamics	Ecological modeling	Wetland Maps
atmospheric conditions	fine scale water level series	vegetation, land use, ...	



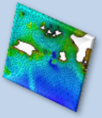
Context

Modeling current and future water level fluctuations in the Great Lakes: temporal evolution of coastal wetlands under various climate scenarios



Climate model
atmospheric conditions

→



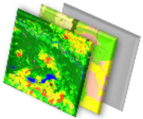
Hydrodynamics
fine scale water level series

Challenging

over the Great Lakes for long periods

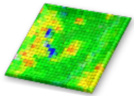
- Computational costs
- Model limitations

→

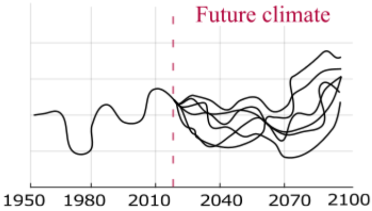


Ecological modeling
vegetation, land use, ...

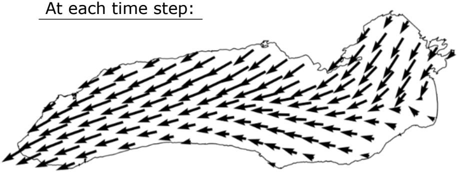
→



Wetland Maps

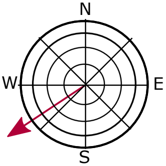


At each time step:



Regional / lake average

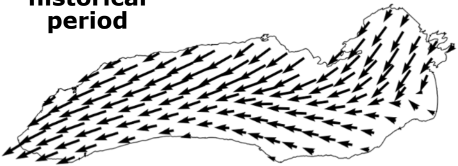
Major wind direction



Average wind speed

Average water level

historical period



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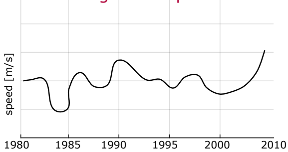
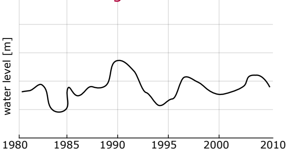
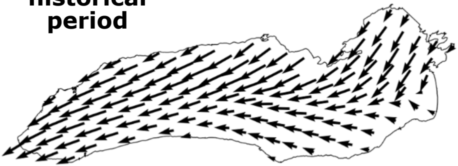


Average water level



historical period



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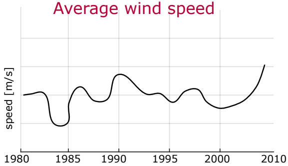
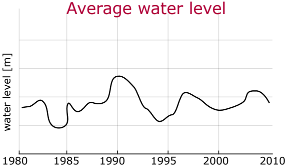
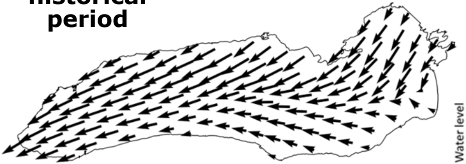


Average water level

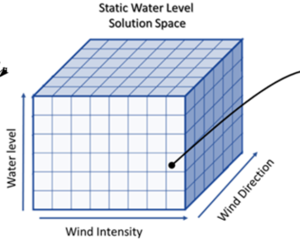


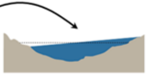
scenarios: direction, speed, and water-level classes

historical period



Regional / lake average

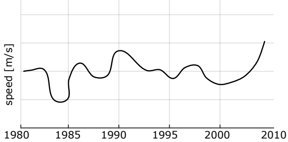




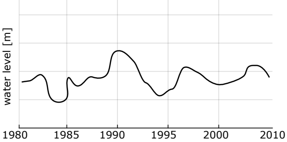
hydrodynamic simulation: 2D water level maps

Major wind direction

Average wind speed



Average water level



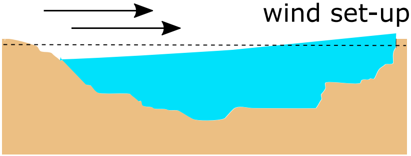
scenarios: direction, speed, and water-level classes

Drawbacks of the scenario approach

Too simplistic scenarios:

Only regional/lake major wind direction and average speed

Only *wind set-up* in one direction



Drawbacks of the scenario approach

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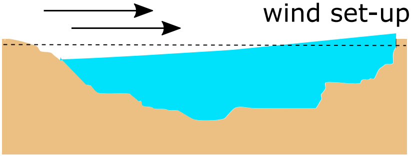
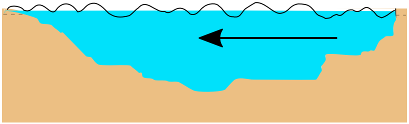
No dynamics in reconstructions:

no *seiche* effect





Drawbacks of the scenario approach

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Still depends on continuous simulations:

Computational burden of the hydrodymical model to produce continuous simulations

proposed solutions

1. Analog method
on PCA projections
of wind fields

Information on wind patterns in scenarios

2. Variational mode
decomposition to construct
seiche scenarios

Information about seiches in reconstructions

3. Reduce the computational cost of simulations by simplifying the dynamical model equations - **avoid scenarios**

Spatial scenarios

Training set: pairs of wind field $(\boldsymbol{u}_t, \boldsymbol{v}_t)$ and water level field $\boldsymbol{h}_t$

Test set: only wind fields $(\boldsymbol{u}_{t'}, \boldsymbol{v}_{t'})$ are available

1. Each wind field $(\boldsymbol{u}_t, \boldsymbol{v}_t)$ from the training set is summarized by low dimensional features $\boldsymbol{z}_t \in \mathbb{R}^d$
2. Project the wind fields $(\boldsymbol{u}_{t'}, \boldsymbol{v}_{t'})$ from the test set onto the same low dimensional space $\boldsymbol{z}_{t'} \in \mathbb{R}^d$
3. For each time step t' of the test set, identify the nearest neighbor t^* in the train set:

$$t^* = \arg\min_t ||\boldsymbol{z}_t - \boldsymbol{z}_{t'}||^2$$

4. Estimate $\boldsymbol{h}_{t'}$ with $\hat{\boldsymbol{h}}_{t'} = \boldsymbol{h}_{t^*}$

Evaluation

Training set: $\{(\boldsymbol{u}_t, \boldsymbol{v}_t), \boldsymbol{h}_t\}_t$
over the period 01/03/1991 - 30/04/1991 (hourly)

Test set: $\{(\boldsymbol{u}_{t'}, \boldsymbol{v}_{t'}), \boldsymbol{h}_{t'}\}_{t'}$
over the period 01/03/1992 - 30/04/1992 (hourly)

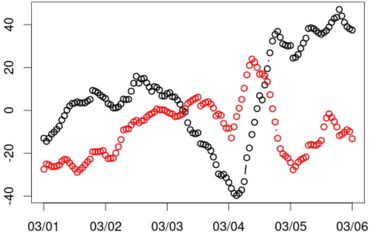
➜ $\boldsymbol{h}_{t'}$ is used for evaluation purposes only

Performance criterion:

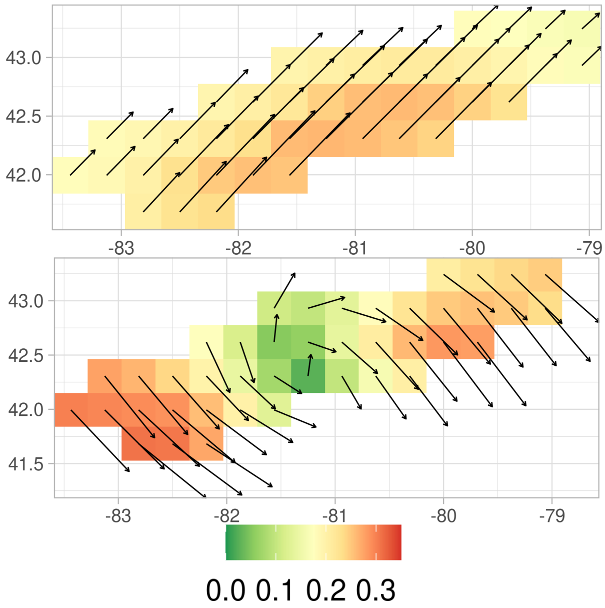
$$\sqrt{\frac{1}{T'} \sum_{t'=1}^{T'} (\boldsymbol{h}_{t'} - \hat{\boldsymbol{h}}_{t'})^2}$$

Wind patterns with PCA

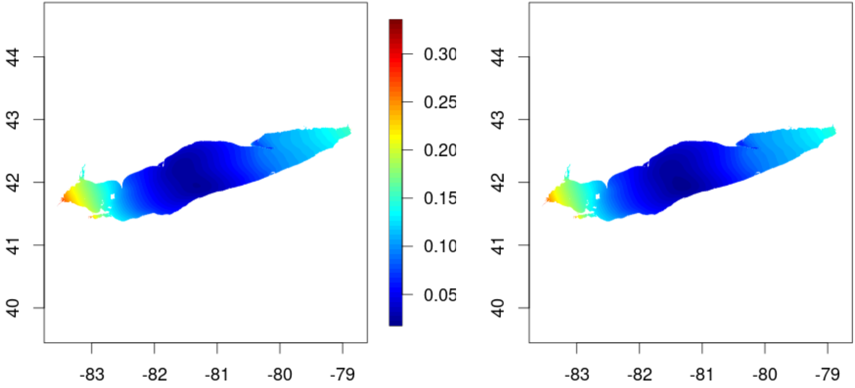
- ➜ Summarize spatial information with Principal Component analysis (PCA)
- ➜ Main wind patterns = principal eigenvectors
  carry the spatial information
- ➜ Time varying features: projections onto first principal eigenvectors
  each wind field is a linear combination of the principal eigenvectors
- ➜ Use temporal features of the wind to identify similar water level fields
  nearest neighbor approach



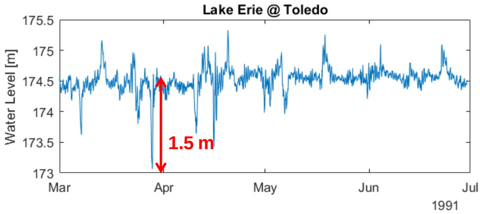
First two principal eivenvectors: intensity and direction

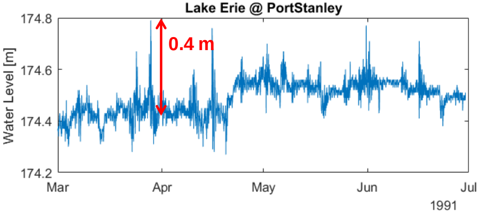


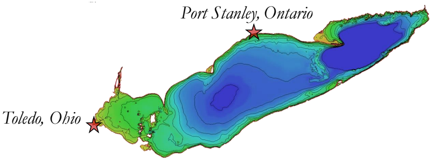
RMSE


44
43
42
41
40
-83
-82
-81
-80
-79
0.30
0.25
0.20
0.15
0.10
0.05
44
43
42
41
40
-83
-82
-81
-80
-79

Analyzed water level signals in Lake Erie



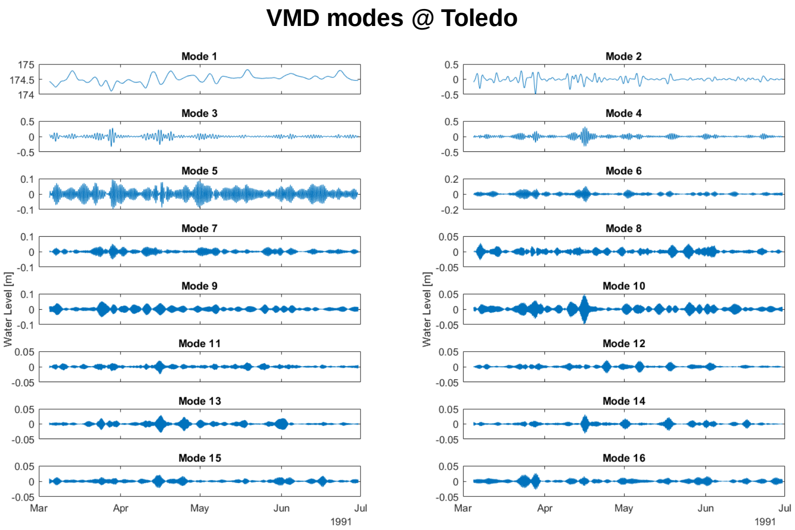


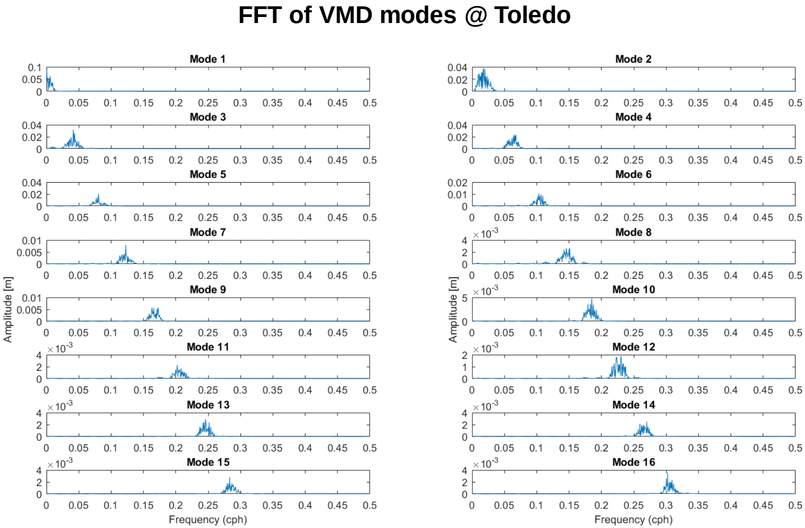


Variational Mode Decomposition (VMD) (Dragomiretskiy & Zosso, 2014) was applied to separate the water level signals into different "modes of variability".

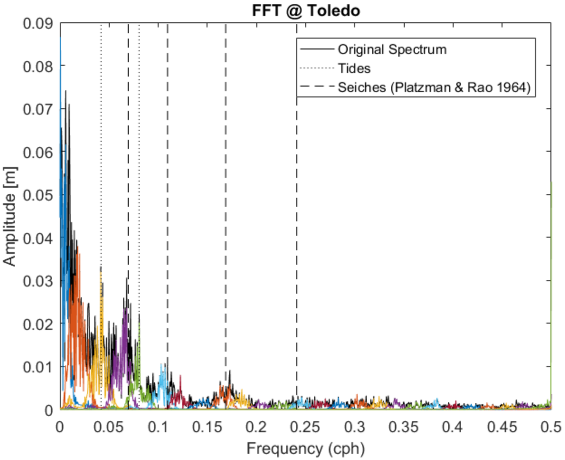
$$\min_{\{u_k\}\{\overline{\omega}_k\}} \left\{ \sum_{k=1}^{K} \left\| \partial_t \left[\left(\delta(t) + \frac{j}{\pi t} \right) * u_k(t) \right] e^{-j\overline{\omega}_k t} \right\|_2^2 \right\}$$

$$s.t. \quad \sum_{k=1}^{K} u_k(t) = H(t)$$

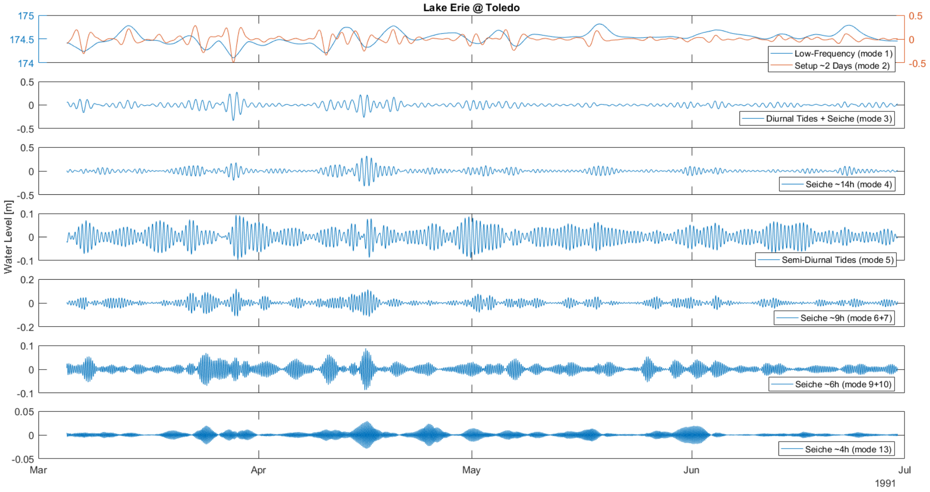

VMD modes @ Toledo
Mode 1
Mode 2
Mode 3
Mode 4
Mode 5
Mode 6
Mode 7
Mode 8
Mode 9
Mode 10
Mode 11
Mode 12
Mode 13
Mode 14
Mode 15
Mode 16
Water Level [m]
Mar
Apr
May
Jun
Jul
1991



FFT of VMD modes @ Toledo
Mode 1
Mode 2
Mode 3
Mode 4
Mode 5
Mode 6
Mode 7
Mode 8
Mode 9
Mode 10
Mode 11
Mode 12
Mode 13
Mode 14
Mode 15
Mode 16
Amplitude [m]
Frequency (cph)



FFT amplitude spectrum for each mode (colors) compared to the original spectrum (black). Vertical lines denote major diurnal and semi-diurnal tidal frequencies (dotted) and documented seiche frequencies (dashed).



Lake Erie @ Toledo
175
174.5
174
0.5
0
-0.5
Low-Frequency (mode 1)
Setup ~2 Days (mode 2)
Diurnal Tides + Seiche (mode 3)
Seiche ~14h (mode 4)
Water Level [m]
0.1
-0.1
Semi-Diurnal Tides (mode 5)
0.2
-0.2
Seiche ~9h (mode 6+7)
Seiche ~6h (mode 9+10)
0.05
-0.05
Seiche ~4h (mode 13)
Mar
Apr
May
Jun
Jul
1991

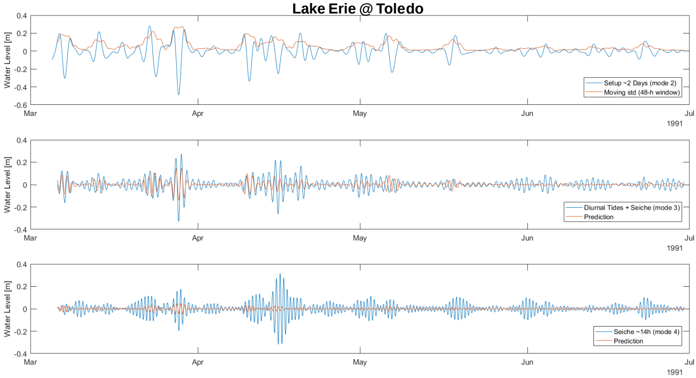
Predicting seiches (high frequencies) from water level setups (low frequencies)?

A simple (linear) regression model was constructed as a preliminary step to building a more complex (non-linear) model, e.g. using machine learning.

$$h_{HF}(t) = \beta_0 + \sum_{j=1}^{N_{modes}} \left(\beta_{1,j}(t)\cos\omega_j t + \beta_{2,j}(t)\sin\omega_j t\right) + \varepsilon$$

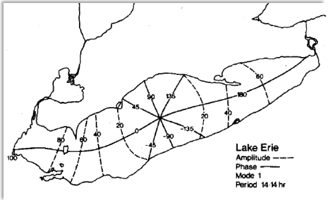
$$\beta_{i,j}(t) = a_{0,i,j} + h_{LF}(t) \times a_{1,i,j}$$

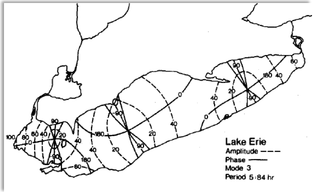
$$A_j(t) = \sqrt{\beta_{1,j}^2 + \beta_{2,j}^2}\,,\ \ \Phi_j(t) = \tan^{-1}\left(\beta_{2,j}/\beta_{1,j}\right)$$



Next steps

- Build a non-linear relationship between setup and seiches, e.g. using machine learning
- Add other variables such as winds or their principal components (from PCA)
- Extend the analysis in 2D using simulated fields
- Represent the seiche phenomenon in a synthetized fashion (e.g. by their amplitudes and phases)
- Preserve the temporal variability in seiche features as opposed to producing static maps





ref. Hamblin (1987)

Reduced System Approach

- Full system being simulated:

$$\frac{\partial h}{\partial t} + \nabla \cdot \boldsymbol{q} = 0,$$

$$\frac{\partial \boldsymbol{q}}{\partial t} + \underbrace{\nabla \cdot \left(\frac{\boldsymbol{q}\boldsymbol{q}^T}{H} \right)}_{\textit{advection}} + \underbrace{c^2 \nabla h}_{\textit{gravity}} - \underbrace{\frac{1}{\rho} \nabla \cdot (H\tau)}_{\substack{\textit{turbulent} \\ \textit{viscosity}}} + \frac{\boldsymbol{\tau}^b}{\rho} - \underbrace{f_c \left(\boldsymbol{q} \times \boldsymbol{e}_z \right)}_{\substack{\textit{Coriolis} \\ \textit{force}}} = \frac{\boldsymbol{\tau}^s}{\rho}$$

- Do we need all these effects to understand Seiche dynamics?
  - Maybe some of the terms are small?

Reduced System Approach: Nondimensionalization

- Factor out the typical sizes by writing

$$\text{dimensional variable'} = \underset{(known)}{\text{typical size}^*} \times \underset{\mathcal{O}(1)}{\text{dimensionless variable}}$$

- for example

$$\text{distance } x'[km] = (\text{length of the lake})^*[km] \times x$$
$$\text{velocity } u'[m/s] = (1m/s) \times u$$

Reduced System Approach: Nondimensionalization

- Using typical magnitudes for Lake Erie, we get

$$\frac{\partial h}{\partial t} + \{10\}\nabla \cdot \boldsymbol{q} = 0,$$

$$\frac{\partial \boldsymbol{q}}{\partial t} + \{1\}\nabla \cdot \left(\frac{\boldsymbol{q}\boldsymbol{q}^T}{H}\right) + \{10\}H\nabla h - \{10^{-8}\}\nabla \cdot (H\tau)$$
$$+\{74\}\boldsymbol{\tau}^b - \{2.3\}(\boldsymbol{q} \times \boldsymbol{e}_z) = \{1.2 \times 10^{-2} w^*\}\boldsymbol{\tau}^s$$

- We can also linearize the remaining system (small waves case):

$$\frac{\partial^2 h}{\partial t^2} + \eta_1 \nabla \cdot (z\nabla h) = 0.$$

Reduced System Approach: Eigenvalue Problem

- The Seiche states are the resonant modes of the system, i.e. solutions to the eigenvalue problem:

$$\eta_1 \nabla \cdot \left(z \nabla h \right) = \lambda h$$

- From the eigenvalues, the Seiche oscillation period is recovered:

$$T = \underbrace{\frac{2L^*}{\sqrt{gz^*}}}_{\substack{Merian's \\ formula}} \sqrt{\frac{\pi^2}{\lambda}}$$

- The eigenvalue problem cab be solved numerically (in a few seconds of computing time). A very preliminary calculation for Lake Erie gives

$$T_1 = 3h\ 7m$$

Conclusions