Anatomy of a Number
March 13–17, 2006

Lecture 1: Small prime gaps: from the Riemann zeta function and pair correlation to the circle method
Lecture 2: Small prime gaps: short divisor sums, correlations, and moments
Lecture 3: Small prime gaps: tuples approximations, Selberg Sieve, and almost primes

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Abstract

Lecture 1:

In this first lecture I will describe how the usual multiplicative approach using zeros of the Riemann zeta function, when applied to the question of finding primes very close together, can make a contribution if one assumes the Riemann Hypothesis and a strong Pair Correlation Conjecture. Without these conjectures little progress can be expected by this approach, and therefore one is forced to turn to additive methods going back to the early days of the circle method as developed by Hardy and Littlewood. The circle method, as shown by Bombieri and Davenport, does lead to unconditional non-trivial results for small gaps between primes, but also encounters a barrier. I will describe some of the sources of this barrier and what information about primes is utilized by the circle method.

Lecture 2:

I will show how the circle method can be replaced by short divisor sums that approximate primes together with a second moment lower bound argument. Gallagher worked out a simple procedure for going from the

Hardy-Littlewood prime tuple conjectures to moments of primes in short intervals, and showed that (under this assumption) the moments are Poisson. With this in mind, Yildirim and I spent three years starting in 1999 trying to find approximations for higher moments and correlations using short divisor sums. I will describe how this naive idea eventually did lead somewhere.

Lecture 3:

I will describe the ideas involved in the work with Pintz and Yildirim that finally did succeed in finds very small gaps between primes, and how this relates to the Selberg sieve and almost primes.