Quantitative Risk Management under Basel III

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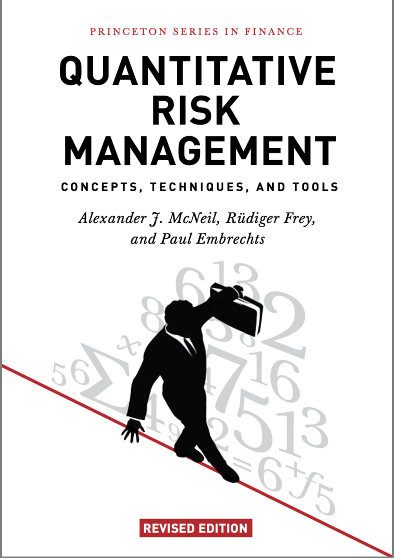

PRINCETON SERIES IN FINANCE
QUANTITATIVE RISK MANAGEMENT
CONCEPTS, TECHNIQUES, AND TOOLS
Alexander J. McNeil, Rüdiger Frey, and Paul Embrechts
REVISED EDITION

Overview

Overview

What is QRM?

For me it is about models:

- Their design;
- Their deployment to model uncertain outcomes in the real world;
- Their statistical estimation or calibration;
- Their interpetation and use in decision making;
- Their continual criticism and refinement.

Valuation models: used to assign values to assets and liabilities that don't have observable prices in a so-called DLT market, with reference to things that do.

Risk management models: used to describe the fluctuations in the values of portfolios of assets/liabilities over future time periods and to determine whether institutions are adequately capitalized.

Unitary models for both only rarely used.

Are QRM Models in Retreat?

- Valuation Models
  - – The boom years for financial mathematicians are over: more standardization in products; generally accepted valuation principles.
  - + There do remain challenges, for example: accounting for market illiquidity, efficient computational methods, pricing counterparty risk for OTC contracts.
- Risk Management Models
  - – In banking the scope to build risk management models has been reduced under Basel III.
  - – Partly a response to the perceived failure of risk models during the financial crisis: “misplaced reliance on sophisticated maths” (Lord Turner, 2009).
  - + In insurance the situation is more positive, certainly as regards Solvency II in Europe, which allows more scope for internal models.
  - + More contracts are traded through CCPs (central counterparties for derivative transactions) so these now have to have regulated risk models.
  - – Critics question whether internal models improve risk and capital management and whether their cost can be justified.

Broad Aims of Basel III

Extend the existing Basel framework in 5 main areas:

1. measures to increase the quality and amount of bank capital by changing the definition of key capital ratios and allowing countercyclical adjustments to these ratios in crises;
2. a strengthening of the framework for counterparty credit risk in derivatives trading with incentives to use central counterparties (exchanges);
3. introduction of a leverage ratio to prevent excessive leverage;
4. introduction of various ratios that ensure that banks have sufficient funding liquidity;
5. measures to force systemically important banks (SIBs) to have even higher capacity to absorb losses.

Full implementation by March 2019.

Some Relevent Points for QRM

- A stated aim is to reduce variability in capital ratios arising from different approaches to modelling at different banks.
- There is clear encouragement to use standardized approaches (rules rather than principles), which have been made more risk sensitive.
- It is proposed, for example, to eliminate the AMA (advanced measurement approach) for operational risk.
- Banks wishing to use internal models will have to meet a series of more stringent qualitative and quantitative standards.
- The importance of model validation and backtesting has increased.

Overview

Balance Sheet of a Bank

Bank XYZ (31st December 2012)			
Assets		**Liabilities**	
Cash (and central bank balance)	£10M	Customer deposits	£80M
Securities	£50M	Bonds issued	
- bonds		- senior bond issues	£25M
- stocks		- subordinated bond issues	£15M
- derivatives		Short-term borrowing	£30M
Loans and mortgages	£100M	Reserves (for losses on loans)	£20M
- corporates			
- retail and smaller clients		Debt (sum of above)	£170M
- government			
Other assets	£20M		
- property			
- investments in companies		Equity	£30M
Short-term lending	£20M		
Total	£200M	Total	£200M

The Trading Book

- Contains assets that are available to trade.
- Can be contrasted with the more traditional banking book which contains loans and other assets that are typically held to maturity and not traded.
- The trading book is supposed to contain assets that are easy to trade, highly liquid and straightforward to value (mark-to-market) at any point in time.
- Examples: fixed income instruments (bonds); certain derivatives.
- The trading book is often identified with market risk whereas the banking book is largely affected by credit risk.
- The Basel rules allow banks to use internal Value-at-Risk (VaR) models to measure market risks in the trading book.
- The trading book was abused in the financial crisis of 2007–2009. Many securitized credit instruments (e.g. CDO tranches) were held in the trading book where they were subject to lower capital requirements.

Changes under Basel III

The new approach to market risk is the result of FRTB - the fundamental review of the trading book. (Basel Committee on Banking Supervision, 2016). Key points are:

- A revised boundary between the trading and banking books, to reduce risk of regulatory arbitrage for less liquid instruments.
- A revised standardized approach (more risk sensitive and granular).
- A revised internal-models approach, with a more rigorous model approval process.
- Change to the expected shortfall (ES) risk measure.
- Incorporation of the risk of market illiquidity, through the introduction of concept of liquidity horizons for risk factors.
- Capital requirements linked to backtesting performance.

The Quantitative Content of FRTB

$$\mathrm{ES}_{h_1}(P,j) = \boxed{h_1\text{-day 97.5\%-ES w.r.t. risk factors with liquidity horizon} \geq h_j}$$

$$\underbrace{\mathrm{ES}_{R,S}}_{\text{reduced risk-factor set; stressed calibration}} = \underbrace{\sqrt{\sum_{j=1}^{5}\left(\sqrt{\frac{h_j - h_{j-1}}{h_1}}\,\mathrm{ES}_{h_1}(P,j)\right)^2}}_{(h_1,h_2,h_3,h_4,h_5)=(10,20,40,60,120),\ h_0=0}$$

$$\mathrm{IMCC}(C) = \mathrm{ES}_{R,S} \times \underbrace{\frac{\mathrm{ES}_{F,C}}{\mathrm{ES}_{R,C}}}_{\frac{\text{full risk-factors; current calibration}}{\text{reduced risk-factors; current calibration}}} \qquad \underbrace{\mathrm{IMCC}(C_i) = \mathrm{ES}_{R,S,i} \times \frac{\mathrm{ES}_{F,C,i}}{\mathrm{ES}_{R,C,i}}}_{\text{standalone calc. for risk factor class } i \text{ (IR,FX,EQ,etc.)}}$$

$$\underbrace{\mathrm{IMCC}}_{\text{calculated daily}} = \rho \cdot \underbrace{\mathrm{IMCC}(C)}_{\text{diversified}} + (1-\rho)\underbrace{\sum_i \mathrm{IMCC}(C_i)}_{\text{undiversified}}, \qquad \rho = 0.5$$

$$C_A = \max\{\mathrm{IMCC}_{t-1} + \underbrace{\mathrm{SES}_{t-1}}_{\text{non-modellable risk factors}},\ \underbrace{m_c}_{\text{multiplier}} \cdot \underbrace{\overline{\mathrm{IMCC}} + \overline{\mathrm{SES}}}_{\text{running averages}}\}$$

$$\mathrm{ACC} = \underbrace{C_A}_{\text{approved desks}} + \underbrace{\mathrm{DRC}}_{\text{default risk charge}} + \underbrace{C_U}_{\text{unapproved desks}}$$

Notes on the Capital Calculation Layers

1. Capital is based on 10-day expected shortfall estimates for the portfolio calculated at the 97.5% level calculated using risk-factor data from a stress period.
2. A square-root-of-time formula takes into account market liquidity risk. Different time horizons may be required to neutralize the risks coming from different risk factors (e.g. by selling out of positions).
3. A scaling process accounts for differences between available data for the stress period and required data for the current portfolio.
4. A further layer of conservatism tries to adjust for overstatement of the diversification between different risk-factor classes.
5. A multiplier, which relates directly to the quality of backtesting results for the whole trading book, is applied to the running average of the capital charge.
6. Desks with poor backtesting results are excepted from the calculation. There are add-ons for non-modellable risks and a default risk charge.

Overview

Expected Shortfall

- Let L denote the negative P&L of a desk or whole trading book. Expected shortfall at level α is the average of losses exceeding the VaR at level α

$$\mathrm{ES}_\alpha(L) = \mathbb{E}(L \mid L \geq \mathrm{VaR}_\alpha(L)),$$

  or the average of VaRs

$$\mathrm{ES}_\alpha(L) = \frac{1}{1-\alpha} \int_{u=\alpha}^{1} \mathrm{VaR}_u(L) \mathrm{d}u$$

  where $\mathrm{VaR}_\alpha(L)$ denotes the α-quantile of distribution of L.
- It captures tail risk better.
- It has better aggregation properties, being subadditive and coherent.
- However, after many years of lobbying for its use, academic have now identified (supposed) shortcomings:
  - a lack of robustness in the estimation procedures (Cont, Deguest, and Scandolo, 2010);
  - lack of a property known as elicitability (Gneiting, 2011; Acerbi and Szekely, 2014).

Elicitability Theory in Brief

- Concept comes from statistical forecasting literature where objective is to forecast a statistical functional $\phi(\cdot)$ describing the loss distribution F_L (Osband, 1985; Gneiting, 2011),
- Suppose there exists a scoring function $S(x, l)$ such that

$$\mathbb{E}\left(S(x, L)\right) = \int_{\mathbb{R}} S(x, l)\mathrm{d}F_L(l)$$

  is minimized by $x = \phi(F_L)$ for loss distribution functions (dfs) $F_L \in \mathcal{L}$.
- Then $\phi(\cdot)$ is said to be an elicitable functional on $\mathcal{L}$ and S is said to be a consistent scoring function for ϕ and $\mathcal{L}$. If the minimum is uniquely attained by $\phi(F_L)$ then S is strictly consistent.
- $\mathbb{E}(L) = \int l\mathrm{d}F_L(l)$ is elicitable for dfs of integrable random variables and the strictly consistent scoring function $S(x, l) = (x - l)^2$.
- $F_L^{-1}(\alpha)$ is elicitable for continuous, strictly increasing dfs and the strictly consistent scoring function

$$S_\alpha(x, l) = |I_{\{l \leq x\}} - \alpha||l - x| \, .$$

Role of Elicitability in Backtesting?

- Given a set of forecasts $x_1, \ldots, x_n$ of an elicitable functional of the loss distribution and a strictly consistent scoring function S, the quality of the estimates can be expressed in the statistic

$$\text{score} = \sum_{t=1}^{n} S(x_t, L_t).$$

- When competing forecasting procedures for the elicitable functional are compared, the score (for large n) will be minimized by the procedure which gives the most accurate forecasts of the functional.
- In the case of VaR forecasts $\widehat{\text{VaR}}_{\alpha,1}, \ldots, \widehat{\text{VaR}}_{\alpha,n}$ the score is given by

$$\text{score} = \sum_{t=1}^{n} S_\alpha(\widehat{\text{VaR}}_{\alpha,t}, L_t) = \sum_{t=1}^{n} |I_{\{L_t > \widehat{\text{VaR}}_{\alpha,t})\}} - (1-\alpha)||L_t - \widehat{\text{VaR}}_{\alpha,t}|.$$

- A bank seeking to minimize the score should always submit its best estimates of VaR at each time point.

Implications of Non-Elicitability of Expected Shortfall

- It is now well known that expected shortfall is not an elicitable functional (Gneiting, 2011; Bellini and Bignozzi, 2013; Ziegel, 2016).
- This has been taken to imply that it is not possible to evaluate the quality of a set of expected shortfall estimates.
- Not true in practice as a number of published tests show (McNeil and Frey, 2000; Acerbi and Szekely, 2017a,b).
- Under a more general theory ES is so-called jointly elicitable with VaR, suggesting the use of score functions for joint forecasts of VaR and ES. (Fissler, Ziegel, and Gneiting, 2016; Fissler and Ziegel, 2015)
- The bank is not penalized or taxed according to the accuracy of its risk measure estimates so the rationale for elicitability is missing.
- The regulatory regime has already taken the decision to separate the choice of risk measure for the capital calculation (ES) from the choice of risk measure for model approval (VaR).

Overview

Concept of Liquidity Horizons

- Risk factors are categorized according to liquidity horizons.

j	h_j	examples
1	10	equity price (large cap), interest rates major currencies, FX rates for major currency pairs
2	20	equity price (small cap), interest rates other currencies, equity price volatility (large cap), credit spread (sovereign)
3	40	credit spread (corporate), FX volatility
4	60	interest rate volatility, equity price volatility (small cap)
5	120	credit spread volatility, certain commodities

- Let $\mathrm{ES}_{h_1}(P, j)$ denote the ES at level 0.975 for the h_1-day (10-day) trading book loss attributable solely to changes in the risk factors with horizon h_j or longer.
- Reflects the practical organisation of risk calculations at most banks: portfolios are expressed in terms of sensitivities (deltas and gammas) to movements in risk factors.

Basel Liquidity Formula

$$\underbrace{\mathrm{ES}_{R,S}}_{\text{reduced risk-factor set; stressed calibration}} = \underbrace{\sqrt{\sum_{j=1}^{5}\left(\sqrt{\frac{h_j - h_{j-1}}{h_1}}\,\mathrm{ES}_{h_1}(P,j)\right)^2}}_{(h_1,h_2,h_3,h_4,h_5)=(10,20,40,60,120),\ h_0=0}$$

- Has to be understood primarily as a rule (or recipe?).
- Must be calibrated to risk-factor change data from a period of stress (S) using reduced set of risk factors (R) for which historical data are available.
- Probably very conservative: liquidity horizons are long; ignores the central limit effect for heavy-tailed risk factors.

Turning the Rule into a Principle

Assumption

(i) The h_1-day risk-factor changes $(\boldsymbol{X}_t)$ form a strict white noise process (an iid process) with mean zero and covariance matrix Σ.

(ii) Each risk factor may be assigned to a unique liquidity bucket B_k defined by a liquidity horizon $h_k \in \mathbb{N}$, $k = 1, \dots, n$.

(iii) The loss (or profit) attributable to risk factors in bucket B_k over any time horizon h with $h/h_1 \in \mathbb{N}$ is given by $\boldsymbol{b}_k' \sum_{t=1}^{\min\{h,h_k\}/h_1} \boldsymbol{X}_t$ where $\boldsymbol{b}_k$ is a weight vector with zeros in any position that corresponds to a risk factor that is not in B_k. (linearity assumption)

- The liquidity formula holds if $(\boldsymbol{X}_t)$ is multivariate Gaussian.
- If $(\boldsymbol{X}_t)$ is multivariate elliptical a more general formula holds:

$$\mathrm{ES}_{R,S} = \eta \sqrt{\sum_{j=1}^{5} \left(\sqrt{\frac{h_j - h_{j-1}}{h_1}} \, \mathrm{ES}_{h_1}(P, j) \right)^2}$$

with η depending on the type of distribution, the portfolio composition $\boldsymbol{b}_1, \dots, \boldsymbol{b}_5$ and Σ, but generally less than 1 (Balter and McNeil, 2017).

Overview

Model Validation Standards

From FRTB:

- "The bank must conduct regular backtesting."
- "Backtesting requirements are based on comparing each desk's 1-day static value-at-risk measure (calibrated to the most recent 12 months' data) at both the 97.5th and 99th percentile."
- "If any given desk experiences either more than 12 exceptions at the 99th or 30 at the 97.5th percentile, all of its positions must be capitalised using the standardised approach."
- "The multiplication factor m_c will be 1.5. Banks must add to this a plus directly related to the ex-post performance of the model."
- "The plus will range from 0 to 0.5 based on the outcome of the backtesting of the bank's daily VaR at the 99th percentile based on the current observations of the full set of risk factors."

$$C_A = \max\{\text{IMCC}_{t-1} + \underbrace{\text{SES}_{t-1}}_{\text{non-modellable risk factors}}, \underbrace{m_c}_{\text{multiplier}} \cdot \underbrace{\overline{\text{IMCC}} + \overline{\text{SES}}}_{\text{running averages}}\}$$

The Multiplier

Let N be the number of exceptions at the $\alpha = 99\%$ level in one trading year of $n = 250$ days and let F_B denote the df of a $B(250, 0.01)$ distribution.

The traffic light system:

$$\begin{aligned} F_B(N) < 0.95 &\quad\Longrightarrow\quad \text{green} \\ F_B(N) \geq 0.95 &\quad\Longrightarrow\quad \text{yellow} \\ F_B(N) \geq 0.9999 &\quad\Longrightarrow\quad \text{red} \end{aligned}$$

This translates to the following thresholds and multipliers:

$$\begin{aligned} N \leq 4 &\quad\Longrightarrow\quad m_c = 1.5 \\ N = 5, 6, 7, 8, 9 &\quad\Longrightarrow\quad m_c = 1.70, 1.76, 1.83, 1.88, 1.92 \\ N \geq 10 &\quad\Longrightarrow\quad m_c = 2, \text{ regulatory intervention} \end{aligned}$$

Standard VaR Backtests Have Low Power

	α	0.975						0.990					
	2-sided	TRUE			FALSE			TRUE			FALSE		
	n | test	Wald	score	LRT	Wald	score	LRT	Wald	score	LRT	Wald	score	LRT
Size	250	5.7	3.9	7.5	2.4	5.0	5.0	8.0	4.0	8.9	1.2	4.0	10.5
	500	7.8	3.9	5.9	2.6	4.7	7.9	12.5	3.7	7.0	1.3	6.7	6.7
	1000	5.0	5.0	4.1	2.8	4.3	6.6	7.5	3.8	5.9	2.7	4.9	8.0
	2000	5.9	5.0	4.2	3.9	5.0	5.0	4.9	5.4	4.1	3.5	5.3	5.3
Power (t5)	250	4.3	4.1	6.9	3.1	6.4	6.4	5.9	17.7	10.7	8.3	17.7	32.4
	500	6.0	5.2	6.5	4.5	7.4	11.3	9.5	22.4	22.8	13.4	33.9	33.9
	1000	4.9	6.9	5.2	5.7	8.0	10.8	17.7	33.0	33.1	33.0	42.7	52.7
	2000	6.0	7.3	5.8	8.3	10.7	10.7	45.3	59.9	52.7	59.9	66.7	66.7
Power (t3)	250	9.7	3.6	10.3	0.8	2.0	2.0	5.6	13.5	9.2	6.0	13.5	26.9
	500	15.8	4.8	9.5	0.6	1.3	2.6	7.8	16.2	16.9	9.3	25.4	25.4
	1000	14.2	9.9	9.7	0.4	0.6	1.0	11.0	22.3	22.5	22.2	30.5	40.5
	2000	25.9	16.6	16.5	0.2	0.3	0.3	27.6	41.4	34.2	41.3	48.8	48.8
Power (st3)	250	4.4	5.4	8.0	4.5	8.6	8.6	10.4	31.2	19.2	18.3	31.2	49.0
	500	6.0	6.9	7.9	6.3	10.1	14.7	22.4	44.2	44.3	31.9	57.2	57.2
	1000	5.5	9.5	6.9	9.0	12.3	16.3	48.6	66.2	66.2	66.2	74.7	82.4
	2000	8.4	12.2	9.8	14.6	17.9	17.9	86.6	92.9	90.1	92.9	95.0	95.0

Estimated size and power of three different types of binomial test (Wald, score, likelihood-ratio test (LRT)) applied to exceptions of the 97.5% and 99% VaR estimates. Results are based on 10000 replications.

Green indicates good results ($\leq$ 6% for the size; $\geq$ 70% for the power); red indicates poor results ($\geq$ 9% for the size; $\leq$ 30% for the power); dark red indicates very poor results ($\geq$ 12% for the size; $\leq$ 10% for the power).

Incentives for Building Dynamic Models

- The estimated one-day 99% VaR (for losses) should be exceeded on 1% of days. This is the property of correct unconditional coverage.
- If a bank uses a dynamic approach that estimates the 99% VaR conditional on all available market data, exceptions should occur independently in time. This is the property of correct conditional coverage.
- A bank that neglects the dynamics of market risk is likely to have clustered VaR exceptions.
- The clustering means that the distribution of annual exceptions has higher variance. There may be more years with more than the expected number of exceptions (and more years with less).
- The higher risk of exceeding the acceptable number of exceptions means a higher risk of a capital multiplier being applied or internal model approval being withdrawn.

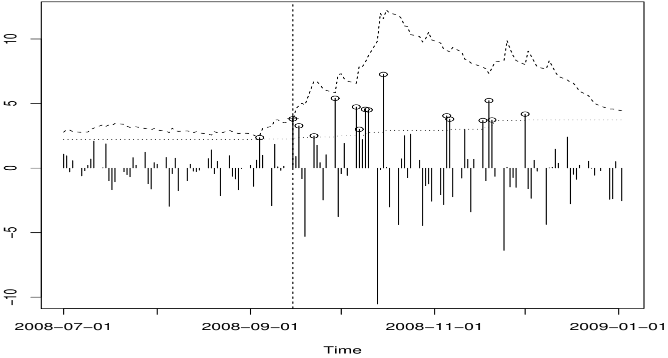
What Models Do Banks Use?

Historical Simulation. Pérignon and Smith (2010) report that 73% of US and international banks use this methodology.

- In essence a non-parametric method based on re-sampling of historical risk-factor changes/returns.
- Sophisticated banks use filtered HS by first applying EWMA volatility estimation and then re-sampling volatility-filtered returns.
- Methods based on the empirical distribution function (re-sampling) may not give good tail models. They can be improved through use of parametric tail models (EVT) but few, if any, banks do this.

Monte Carlo Method. A minority of banks build parametric models for market risk factors and analyse trading book P&L under randomly generated (Monte Carlo) scenarios.

Illustrative Picture (an extreme S&P episode)



Dotted line is HS; dashed line is dynamic method; vertical line is Lehmann. Circle is VaR exception for HS; cross is VaR exception for dynamic method.

Overview

Summary

- Internal risk models are in retreat in banking.
- In the trading book significant changes to the modelling requirements have been introduced (expected shortfall, liquidity horizons).
- The approach is very rules-based with circumscribed use of an internal model.
- Model validation (backtesting) has become more stringent and now extends to desk level.
- Backtesting exceptions may lead to a higher multiplier applied to a firm's capital requirement and may also lead to withdrawal of internal model status for particular desks or the whole trading book.
- If risk is not modelled dynamically (as is the case for historical simulation) the variance of the distribution of the number of annual exceptions is high, even if the expected value may be correct.
- A bank that devotes resource to improving statistical modelling of risk factors should be awarded with lower capital.

Links to my Research

While the Basel capital regime is based on backtesting the VaR risk measure, more powerful tests of the tail of the trading book model are available including:

- multinomial tests of exception counts (Kratz, Lok, and McNeil, 2016);
- tests based on PIT-values (probability-integral transform) in the tail (Gordy, Lok, and McNeil, 2017);
- dynamic variants of these to test independence.

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