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Elliptic hypergeometric integrals

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Abstract

Euler's beta (and gamma) integral and the associated orthogonal polynomials lie at the core of much of the theory of special functions, and many generalizations have been studied, including multivariate analogues (the Selberg integral; also work of Dixon and Varchenko), q-analogues (Askey–Wilson, Nasrallah–Rahman), and both (work of Milne–Lilly and Gustafson; Macdonald and Koornwinder for orthgonal polynomials). (Among these are the more tractable sums arising in random matrices/tilings/etc.) In 2000, van Diejen and Spiridonov conjectured a further generalization of the Selberg integral, going beyond q to the elliptic level (replacing q by a point on an elliptic curve). I'll discuss two proofs of their conjecture, and the corresponding elliptic analogue of the Macdonald and Koornwinder orthogonal polynomials. In addition, I'll discuss a further generalization of the elliptic Selberg with a (partial) symmetry under the exceptional Weyl group E_8, and its relation to Sakai's elliptic Painlevé equation.