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Quantum Many-body Systems and the Nonlinear Schrödinger Equation

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At extremely cold temperatures there forms a new state of matter, called Bose–Einstein condensation, with weird behavior : quantum effects are visible macroscopically, and friction no longer matters. Certain aspects of this phenomenon are nicely understood by scaling limits.

We describe two scaling limits for systems of many quantum particles : mean-field systems and Bose–Einstein condensation. First, in mean-field systems, the microscopic particles experience weak and diffuse interactions, and the Hartree equation turns out to describe them macroscopically. Second, in Bose–Einstein condensation (which can be viewed as a limiting case of mean-field systems), the particles experience strong and short-scale interactions, and the cubic nonlinear Schrödinger equation turns out to describe it macroscopically.

In joint work with Benjamin Schlein and Gigliola Staffilani, we have handled the two-dimensional Bose–Einstein condensation–and the periodic case is especially interesting, as it involves some techniques from analytic number theory.