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## Series Jackson Networks and Non-crossing Probabilities

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In this talk, we study a class of queueing systems known as series Jackson networks. The queue-length process of these networks is Markov, but the inherently complex dependencies across stations in the network pose a challenge to analyze its time-dependent behavior. We show that its transition probabilities can be written as a finite sum of non-crossing probabilities, so that questions on time-dependent queueing behavior are translated to questions on non-crossing probabilities. To illustrate the use of this connection, we prove that the relaxation time (i.e., the reciprocal of the ‘spectral gap’) of a positive recurrent system equals the relaxation time of an M/M/1 queue with the same arrival and service rates as the network’s bottleneck station. This resolves a 1985 conjecture of Blanc, which had only been proved for two queues in series.

*This is joint work with Jon Warren, University of Warwick.*