Staffing and Scheduling in Multiskill and Blend Call Centers

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(Joint work with Tolga Cezik, Éric Buist, and Thanos Avramidis)

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- Ongoing and future work. Conclusion.

Multiskill Agents and Skill-Based Routing

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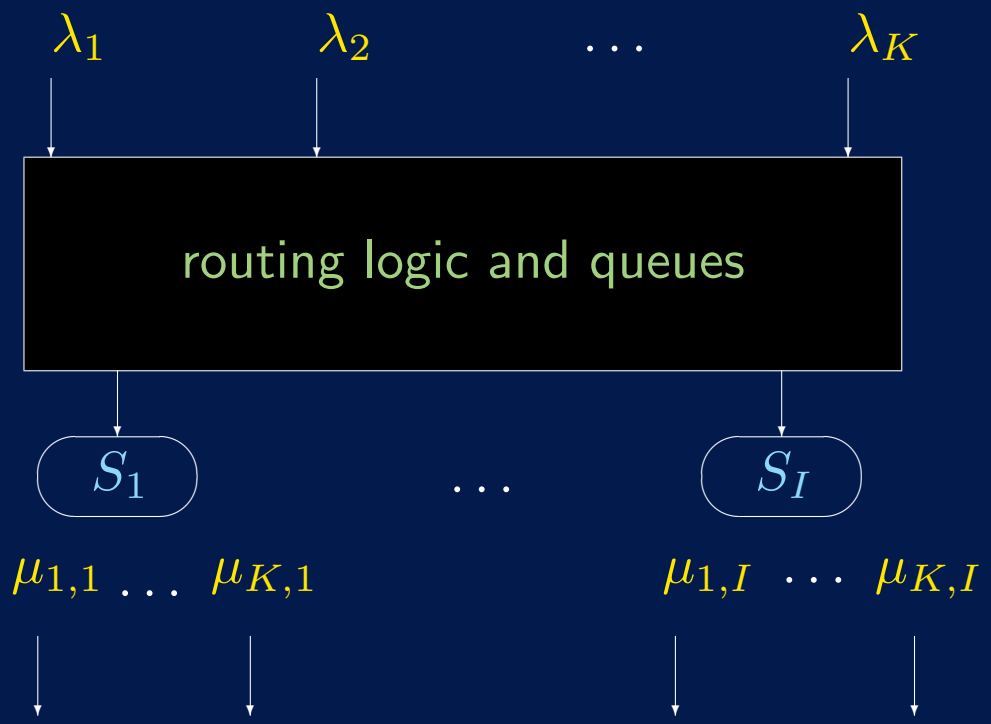

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Performance measures and constraints:
Total cost of agents, quality of service (QoS) (e.g., fraction of calls answered within 20 seconds, abandonment ratio, etc.).

Skill-based routing (SBR) strategies:

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Here, we assume that the routing rules are fixed; we do not optimize them.

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Integer Programming Formulation

Call types	$k = 1, \ldots, K$;
Agent types (or skill groups)	$i = 1, \ldots, I$;
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We have $\mathbf{y} = \mathbf{A}\mathbf{x}$ where $\mathbf{A}$ is block diagonal with i blocks $\tilde{\mathbf{A}}$, and element (p, q) of $\tilde{\mathbf{A}}$ is 1 if shift q covers period p, 0 otherwise.

$g_{k,p}(\mathbf{y})$ = long-run QoS for call type k in period p.
E.g., fraction of calls answered within 20 seconds, or $s_{k,p}$ seconds:

$$g_{k,p}(\mathbf{y}) = \frac{E[\text{num. of calls answered within the limit in period } p]}{E[\text{num. of calls in period } p]}.$$

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  We are developing a Java library for the simulation of call centers.

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\text{subject to} \quad & \mathbf{A}\mathbf{x} = \mathbf{y}, \\
& g_{k,p}(\mathbf{y}) \geq l_{k,p} \qquad \text{for all } k, p, \\
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This does not give an optimal solution to the original scheduling problem.
And the gap could be quite significant.

Steady-state approximations

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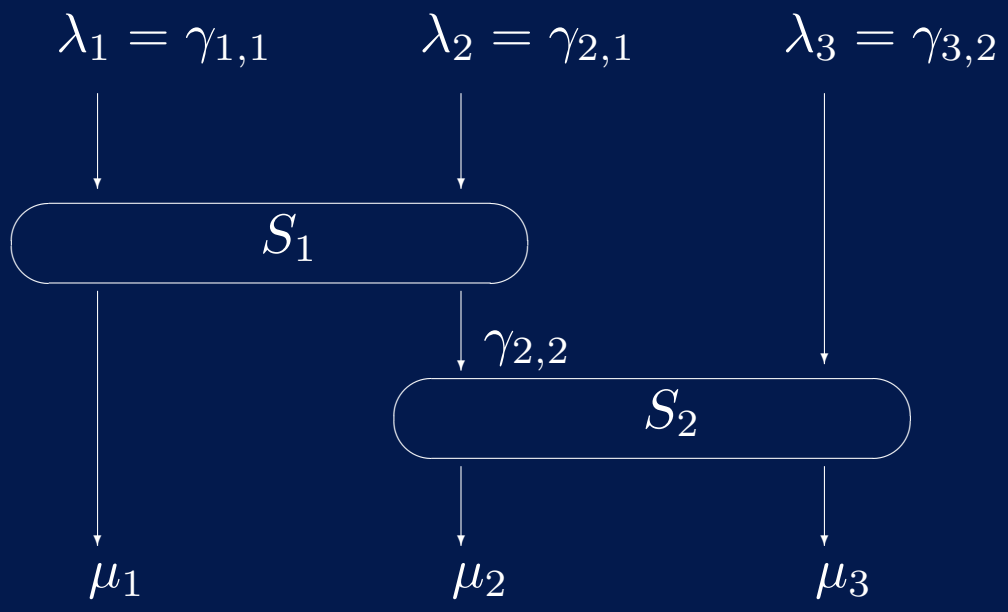
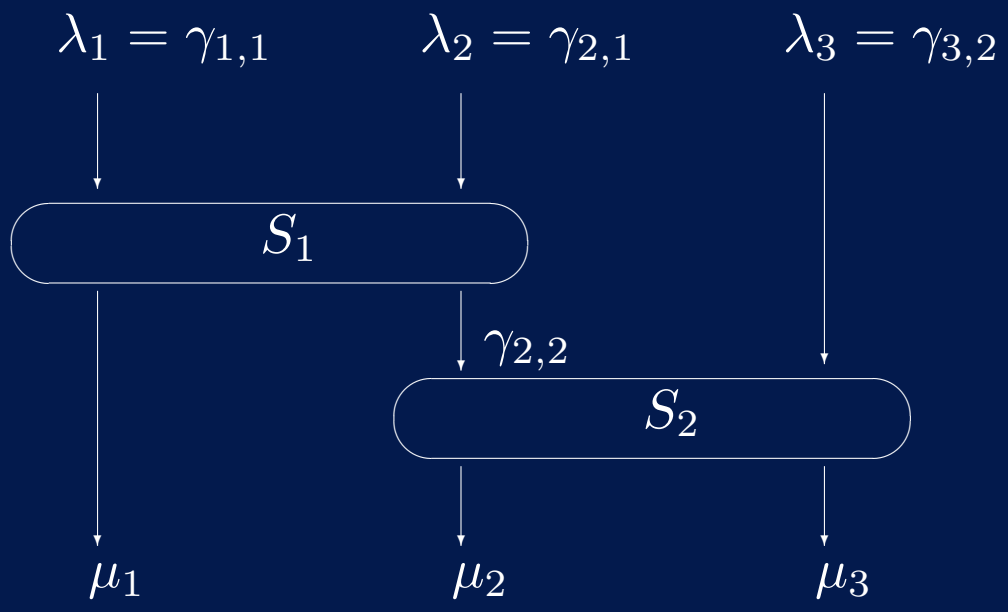


$\gamma_{k,i}$ = net arrival rate of call type k at skill group i;



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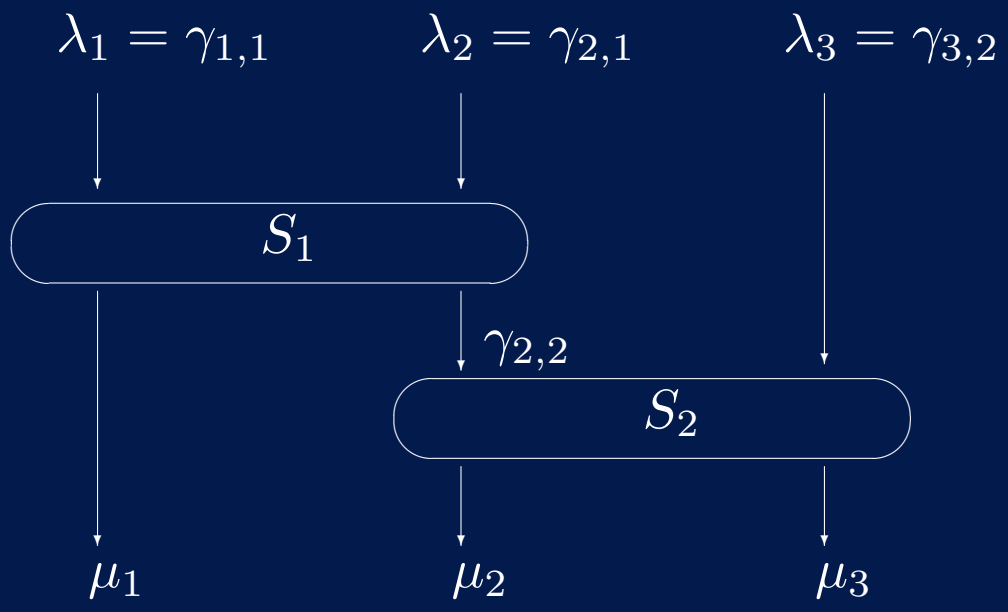

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$a_i = \sum_k \gamma_{k,i}/\mu_{k,i}$ = load at skill group i;

System of nonlinear equations:

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\gamma_{k,\mathrm{first}(k)} &= \lambda_k, \\
\gamma_{k,\mathrm{succ}(k,i)} &= \gamma_{k,i} B(y_i, a_i) \qquad \text{whenever } \mathrm{succ}(k,i) \text{ exists,} \\
B(y_i, a_i) &= \frac{a_i^{y_i}/y_i!}{1 + a_i + \cdots + a_i^{y_i}/y_i!} \qquad \text{(blocking probability for skill group } i\text{).}
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System of nonlinear equations:

$$\begin{aligned}
\gamma_{k,\mathrm{first}(k)} &= \lambda_k, \\
\gamma_{k,\mathrm{succ}(k,i)} &= \gamma_{k,i} B(y_i, a_i) \qquad \text{whenever } \mathrm{succ}(k,i) \text{ exists}, \\
B(y_i, a_i) &= \frac{a_i^{y_i}/y_i!}{1 + a_i + \cdots + a_i^{y_i}/y_i!} \qquad \text{(blocking probability for skill group } i\text{)}.
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There are also better approximations for the loss rate.

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To compute them at different values of $\mathbf{y}$, we simply use simulation with common random numbers.

Empirical scheduling optimization problem (sample version of the problem):

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\begin{aligned}
\min \quad & \mathbf{c}^{\mathrm{t}}\mathbf{x} = \sum_{i=1}^{I} \sum_{q=1}^{Q} c_{i,q} x_{i,q} \\
\text{subject to} \quad & \mathbf{A}\mathbf{x} = \mathbf{y}, \\
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Similar formulation for the staffing problem.

We know that $G_{n,k,p}(\mathbf{y})$ converges to $g_{k,p}(\mathbf{y})$ for each (k, p) and each $\mathbf{y}$. Thus, the empirical problem converges to the exact problem when $n \longrightarrow \infty$.

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Theorem (follows from Vogel 1994; see also Atlason et al. 2004).
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$$P[\mathcal{Y}_n^* = \mathcal{Y}^*] \geq 1 - \alpha e^{-\beta n}.$$

Solving the optimization problems

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— extend the methodology to the blend and multiskill settings;
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We explain the algorithm for the exact functions $g_\bullet$, but it works the same way if they are replaced by approximations or estimation $G_{n,\bullet}$.

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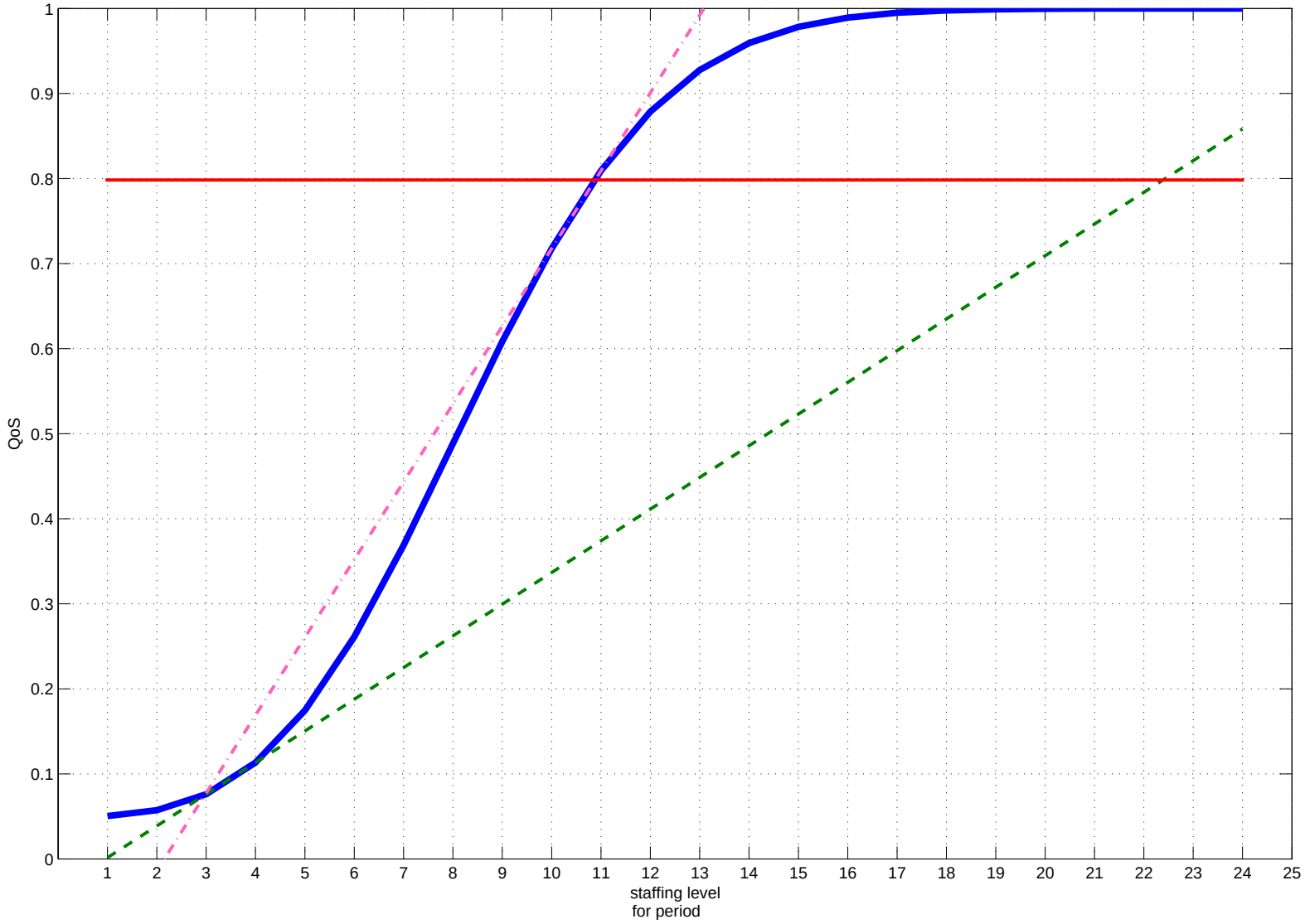
where $\bar{\mathbf{q}}$ is any subgradient of g at $\bar{\mathbf{y}}$.


QoS
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Adding this linear cut inequality to the constraints removes $\bar{\mathbf{y}}$ from the current set of feasible solutions.

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Heuristic: simply use forward finite differences.
Compute g at $\mathbf{y}$ and at $\mathbf{y} + \mathbf{e}_j$ for $j = 1, \ldots, IP$.
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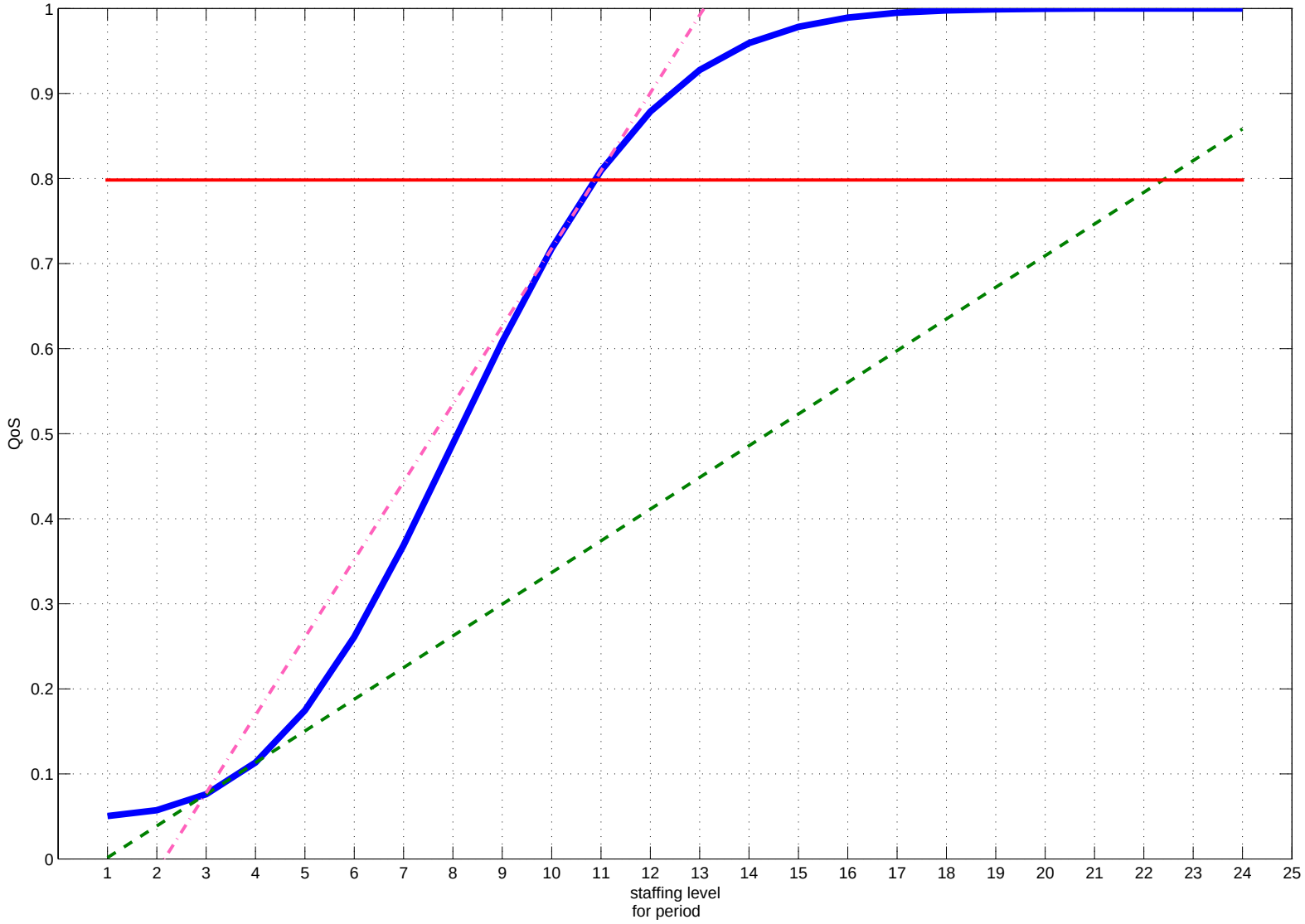
What should we do when we detect non-concavity?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That is, impose $\mathbf{y} \in \mathcal{Y}$, in which the $g_\bullet$ are more likely to be concave.

What we do: impose that for each period p, the skill supply of the available agents can cover the total load for each call type.

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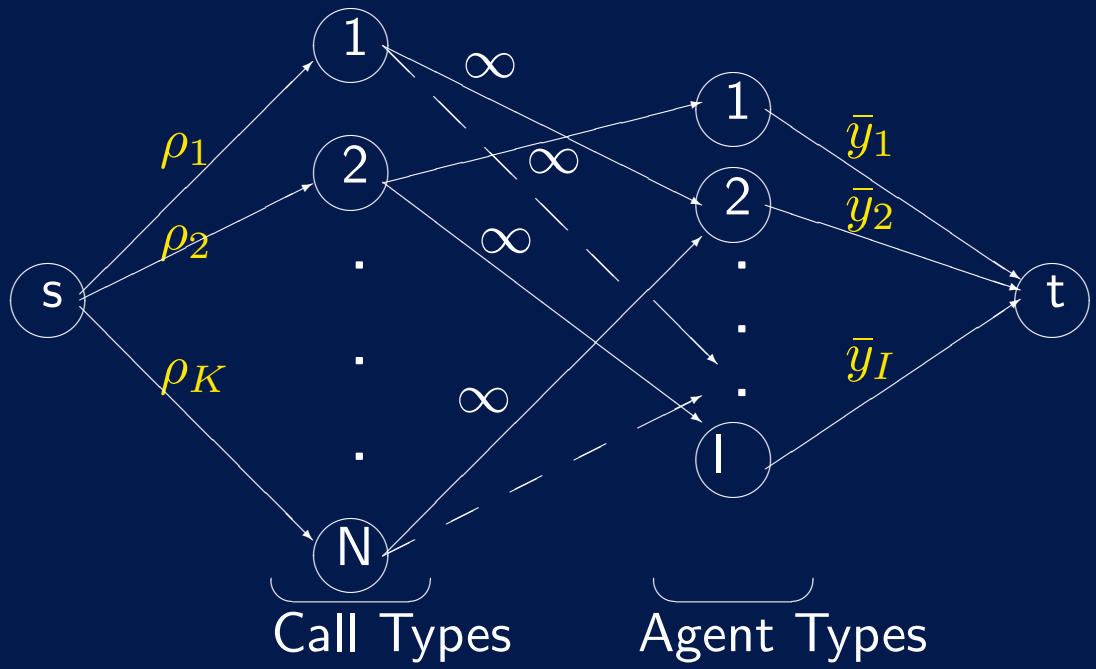
That is, if $\rho_k = \lambda_k/\mu_k$ is the load for call type k (assuming the service rate depends on call type only), the maximum flow in the following graph must be $\sum_{k=1}^{K} \rho_k$:



In this graph, there is an arc from call type k to agent type i if and only if agent type i can handle call type k.

This "stability" condition is not strictly necessary when there are losses (or abandonments). And if all calls must be served, it may not be sufficient for stability (depending, e.g., on priority rules). This is only a heuristic to cut out areas of high non-concavity of the functions g_*.

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Cut-generation method with queueing approx. can be problematic because we must handle low-QoS solutions.

Computational Experiments

Optimization with simulation (sample problem).
Single period, assume steady-state (start the system full and use batch means with 20 batches to estimate variance + 1 additional batch for warmup).

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Routing: each call type has an ordered list of agents types, and goes in a queue (per call type) if all these agents are busy.
Each agent type has an ordered list of call types, for when he becomes free.

Example 1a (partly from Koole and Talim 2000).
5 call types, 12 agent types.

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Skill groups and routing:

k	agent type i											
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This makes the system highly unbalanced $\Rightarrow$ generally more difficult to solve.

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3		2		4		6	7		9	10	11	12
4					5					10		12
5								8	9	10	11	12

An agent that becomes free looks at call queues in numeric order.

This makes the system highly unbalanced $\Rightarrow$ generally more difficult to solve.

Agent's costs: $1 + \kappa \cdot$(number of skills -1), where $\kappa = 0.10$, 0.15, 0.20.

Arrival rate $\lambda_k = 240$ per hour for all k;
Service rate $\mu_k = 12$ per hour for all k.

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No abandonment.

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No abandonment.

Minimal QoS overall: $l = 0.80$.

Example 1a. $\ell = .80$ and $\ell_k = 0$ for all k.

κ cuts	obj. CPU (sec)	QoS QoS KT	QoS per call type staffing vector
500 hours			
.10	119.6	$.803 \pm .002$	(.98, .97, .99, .79, .00)
10	746	.540	(0, 18, 0, 0, 20, 20, 27, 0, 0, 21, 0, 0)
.15	127.7	$.802 \pm .002$	(.98, .99, .99, .82, .00)
16	1387	.624	(1, 0, 0, 17, 20, 37, 11, 0, 0, 21, 0, 0)
.20	131.8	$.800 \pm .002$	(.98, .96, .99, .80, .00)
17	1500	.491	(0, 15, 0, 4, 20, 22, 23, 0, 0, 21, 0, 0)

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16	1387	.624	(1, 0, 0, 17, 20, 37, 11, 0, 0, 21, 0, 0)
.20	131.8	$.800 \pm .002$	(.98, .96, .99, .80, .00)
17	1500	.491	(0, 15, 0, 4, 20, 22, 23, 0, 0, 21, 0, 0)
50 hours			
.10	125.7	$.800 \pm .005$	(1.00, .98, 1.00, .88, .00)
10	68	.844	(6, 24, 0, 9, 20, 22, 12, 0, 0, 21, 0, 0)
.15	142.7	$.827 \pm .025$	(1.00, .80, 1.00, .98, .35)
19	125	.959	(0, 29, 21, 18, 26, 9, 0, 0, 0, 22, 0, 0)
.20	141.6	$.806 \pm .028$	(1.00, .99, .99, .80, .25)
13	85	.906	(0, 22, 0, 8, 32, 20, 12, 21, 1, 0, 0, 0)

Constraints per call type: $\ell = .80$ and $\ell_k = 0.5$ for all k.

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So in this case (which typically happens for highly unbalanced systems), we need a different type of heuristic.

We add a non-subgradient-type cut as follows:
Choose k for which $G_{n,k}(\bar{\mathbf{y}}) - \ell_k$ is smallest and replace the constraint

$$\sum_{i=1}^{I} y_i \mathcal{I}[\text{agent } i \text{ has skill } k] \geq \rho_k$$

by

$$\sum_{i=1}^{I} y_i \mathcal{I}[\text{agent } i \text{ has skill } k] \geq (1+\delta)\rho_k$$

for some $\delta > 0$.

Example 1a. $\ell = .80$ and $\ell_k = 0.50$ for all k, 500 hours.

κ cuts	obj. CPU (sec)	QoS QoS KT	QoS per call type staffing vector
.10	124.5	$.809 \pm .006$	(.99, .79, .86, .75, .63)
24	222	.701	(0, 5, 0, 0, 21, 0, 0, 43, 10, 24, 2, 1)

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24	222	.701	(0, 5, 0, 0, 21, 0, 0, 43, 10, 24, 2, 1)
.15	133.0	$.800 \pm .003$	(.97, .72, .92, .79, .58)
18	207	.633	(0, 6, 0, 3, 17, 0, 0, 45, 0, 35, 0, 0)

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.15	133.0	$.800 \pm .003$	(.97, .72, .92, .79, .58)
18	207	.633	(0, 6, 0, 3, 17, 0, 0, 45, 0, 35, 0, 0)
.20	142.2	$.818 \pm .007$	(.98, .70, .90, .84, .66)
22	220	.677	(9, 2, 0, 0, 16, 0, 0, 39, 0, 41, 0, 0)

Example 1b. Same, except: Abandonment with prob. 0.01, then at rate 10.0.
Arrival rates $\lambda_1 = \lambda_3 = \lambda_5 = 440$, $\lambda_2 = \lambda_4 = 540$. $\ell = .80$.
Simulate 500 operating hours.

κ cuts	obj. CPU (sec)	QoS	QoS per call type staffing vector
$\ell_k = 0$			
.10	218.5	$.800 \pm .003$	(.99, .90, .97, .90, .15)
13	1521		(34, 31, 15, 0, 45, 37, 0, 12, 0, 27, 0, 0)

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.15 13	227.25 1521	$.802 \pm .002$	(.99, .90, .97, .89, .18) (33, 34, 30, 0, 43, 20, 0, 15, 0, 26, 0, 0)

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.20 13	236.0 1530	$.800 \pm .003$	(.99, .90, .97, .89, .18) (33, 34, 30, 0, 43, 20, 0, 15, 0, 26, 0, 0)
$\ell_k = 0.5$			
.10 24	221.6 1474	$.801 \pm .006$	(0.99, 0.94, 0.86, 0.67, 0.52) (20, 31, 36, 0, 42, 4, 0, 50, 0, 17, 0, 0)

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.15 24	232.5 1475	$.803 \pm .007$	(0.99, 0.94, 0.88, 0.66, 0.52) (25, 33, 34, 0, 40, 2, 0, 50, 0, 17, 0, 0)

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.15 24	232.5 1475	$.803 \pm .007$	(0.99, 0.94, 0.88, 0.66, 0.52) (25, 33, 34, 0, 40, 2, 0, 50, 0, 17, 0, 0)
.20 24	242.6 1410	$.800 \pm .006$	(0.99, 0.96, 0.81, 0.67, 0.53) (25, 34, 41, 0, 45, 4, 0, 46, 0, 8, 0, 0)

How the algorithm behaves:

Example 1b, 500 hours, $\kappa = 0.20$, only overall QoS constraint.

iteration	obj.	QoS per call type	QoS	staffing vector
1. flow-cuts	233.0	(.99, .97, .60, .55, .25)	.685	(36, 36, 45, 0, 45, 1, 0, 37, 0, 0, 0, 0)

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2. QoS cut	233.0	(.99, .96, .94, .50, .18)	.721	(36, 36, 16, 0, 45, 30, 0, 37, 0, 0, 0, 0)

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3. QoS cut	233.0	—	—	(36, 36, 0, 0, 45, 46, 0, 19, 0, 18, 0, 0)
flow-cuts	233.0	(.99, .90, .98, .84, .08)	.773	(36, 36, 1, 0, 45, 45, 0, 19, 0, 18, 0, 0)

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flow-cuts	233.0	(.99, .90, .98, .84, .08)	.773	(36, 36, 1, 0, 45, 45, 0, 19, 0, 18, 0, 0)
4. QoS cut	235.8	(.99, .97, .95, .75, .14)	.772	(36, 23, 0, 0, 45, 58, 0, 26, 0, 12, 0, 0)

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5. QoS cut	235.8	(.99, .94, .97, .89, .10)	.793	(36, 22, 1, 0, 45, 59, 0, 9, 0, 28, 0, 0)

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6. QoS cut	235.8	(.99, .86, .97, .93, .11)	.786	(34, 24, 1, 0, 47, 57, 0, 0, 0, 37, 0, 0)

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6. QoS cut	235.8	(.99, .86, .97, .93, .11)	.786	(34, 24, 1, 0, 47, 57, 0, 0, 0, 37, 0, 0)
7. QoS cut	236.0	(.99, .96, .96, .78, .14)	.778	(22, 35, 3, 0, 58, 45, 0, 33, 0, 4, 0, 0)

How the algorithm behaves:

Example 1b, 500 hours, $\kappa = 0.20$, only overall QoS constraint.

iteration	obj.	QoS per call type	QoS	staffing vector
1. flow-cuts	233.0	(.99, .97, .60, .55, .25)	.685	(36, 36, 45, 0, 45, 1, 0, 37, 0, 0, 0, 0)
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7. QoS cut	236.0	(.99, .96, .96, .78, .14)	.778	(22, 35, 3, 0, 58, 45, 0, 33, 0, 4, 0, 0)
8. QoS cut	236.0	(.99, .95, .98, .86, .12)	.795	(28, 29, 5, 0, 49, 52, 0, 15, 0, 22, 0, 0)

How the algorithm behaves:

Example 1b, 500 hours, $\kappa = 0.20$, only overall QoS constraint.

iteration	obj.	QoS per call type	QoS	staffing vector
1. flow-cuts	233.0	(.99, .97, .60, .55, .25)	.685	(36, 36, 45, 0, 45, 1, 0, 37, 0, 0, 0, 0)
2. QoS cut	233.0	(.99, .96, .94, .50, .18)	.721	(36, 36, 16, 0, 45, 30, 0, 37, 0, 0, 0, 0)
3. QoS cut	233.0	—	—	(36, 36, 0, 0, 45, 46, 0, 19, 0, 18, 0, 0)
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4. QoS cut	235.8	(.99, .97, .95, .75, .14)	.772	(36, 23, 0, 0, 45, 58, 0, 26, 0, 12, 0, 0)
5. QoS cut	235.8	(.99, .94, .97, .89, .10)	.793	(36, 22, 1, 0, 45, 59, 0, 9, 0, 28, 0, 0)
6. QoS cut	235.8	(.99, .86, .97, .93, .11)	.786	(34, 24, 1, 0, 47, 57, 0, 0, 0, 37, 0, 0)
7. QoS cut	236.0	(.99, .96, .96, .78, .14)	.778	(22, 35, 3, 0, 58, 45, 0, 33, 0, 4, 0, 0)
8. QoS cut	236.0	(.99, .95, .98, .86, .12)	.795	(28, 29, 5, 0, 49, 52, 0, 15, 0, 22, 0, 0)
9. QoS cut	236.0	(.99, .91, .97, .91, .12)	.796	(22, 35, 12, 0, 59, 35, 0, 23, 0, 14, 0, 0)

How the algorithm behaves:

Example 1b, 500 hours, $\kappa = 0.20$, only overall QoS constraint.

iteration	obj.	QoS per call type	QoS	staffing vector
1. flow-cuts	233.0	(.99, .97, .60, .55, .25)	.685	(36, 36, 45, 0, 45, 1, 0, 37, 0, 0, 0, 0)
2. QoS cut	233.0	(.99, .96, .94, .50, .18)	.721	(36, 36, 16, 0, 45, 30, 0, 37, 0, 0, 0, 0)
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5. QoS cut	235.8	(.99, .94, .97, .89, .10)	.793	(36, 22, 1, 0, 45, 59, 0, 9, 0, 28, 0, 0)
6. QoS cut	235.8	(.99, .86, .97, .93, .11)	.786	(34, 24, 1, 0, 47, 57, 0, 0, 0, 37, 0, 0)
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8. QoS cut	236.0	(.99, .95, .98, .86, .12)	.795	(28, 29, 5, 0, 49, 52, 0, 15, 0, 22, 0, 0)
9. QoS cut	236.0	(.99, .91, .97, .91, .12)	.796	(22, 35, 12, 0, 59, 35, 0, 23, 0, 14, 0, 0)
1. QoS cut	236.0	(.99, .94, .97, .87, .12)	.795	(26, 31, 8, 0, 52, 46, 0, 17, 0, 20, 0, 0)

How the algorithm behaves:

Example 1b, 500 hours, $\kappa = 0.20$, only overall QoS constraint.

iteration	obj.	QoS per call type	QoS	staffing vector
1. flow-cuts	233.0	(.99, .97, .60, .55, .25)	.685	(36, 36, 45, 0, 45, 1, 0, 37, 0, 0, 0, 0)
2. QoS cut	233.0	(.99, .96, .94, .50, .18)	.721	(36, 36, 16, 0, 45, 30, 0, 37, 0, 0, 0, 0)
3. QoS cut	233.0	—	—	(36, 36, 0, 0, 45, 46, 0, 19, 0, 18, 0, 0)
flow-cuts	233.0	(.99, .90, .98, .84, .08)	.773	(36, 36, 1, 0, 45, 45, 0, 19, 0, 18, 0, 0)
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5. QoS cut	235.8	(.99, .94, .97, .89, .10)	.793	(36, 22, 1, 0, 45, 59, 0, 9, 0, 28, 0, 0)
6. QoS cut	235.8	(.99, .86, .97, .93, .11)	.786	(34, 24, 1, 0, 47, 57, 0, 0, 0, 37, 0, 0)
7. QoS cut	236.0	(.99, .96, .96, .78, .14)	.778	(22, 35, 3, 0, 58, 45, 0, 33, 0, 4, 0, 0)
8. QoS cut	236.0	(.99, .95, .98, .86, .12)	.795	(28, 29, 5, 0, 49, 52, 0, 15, 0, 22, 0, 0)
9. QoS cut	236.0	(.99, .91, .97, .91, .12)	.796	(22, 35, 12, 0, 59, 35, 0, 23, 0, 14, 0, 0)
1. QoS cut	236.0	(.99, .94, .97, .87, .12)	.795	(26, 31, 8, 0, 52, 46, 0, 17, 0, 20, 0, 0)
11. QoS cut	236.2	(.99, .94, .98, .88, .12)	.798	(29, 28, 5, 0, 48, 52, 0, 11, 0, 27, 0, 0)

How the algorithm behaves:

Example 1b, 500 hours, $\kappa = 0.20$, only overall QoS constraint.

iteration	obj.	QoS per call type	QoS	staffing vector
1. flow-cuts	233.0	(.99, .97, .60, .55, .25)	.685	(36, 36, 45, 0, 45, 1, 0, 37, 0, 0, 0, 0)
2. QoS cut	233.0	(.99, .96, .94, .50, .18)	.721	(36, 36, 16, 0, 45, 30, 0, 37, 0, 0, 0, 0)
3. QoS cut	233.0	—	—	(36, 36, 0, 0, 45, 46, 0, 19, 0, 18, 0, 0)
flow-cuts	233.0	(.99, .90, .98, .84, .08)	.773	(36, 36, 1, 0, 45, 45, 0, 19, 0, 18, 0, 0)
4. QoS cut	235.8	(.99, .97, .95, .75, .14)	.772	(36, 23, 0, 0, 45, 58, 0, 26, 0, 12, 0, 0)
5. QoS cut	235.8	(.99, .94, .97, .89, .10)	.793	(36, 22, 1, 0, 45, 59, 0, 9, 0, 28, 0, 0)
6. QoS cut	235.8	(.99, .86, .97, .93, .11)	.786	(34, 24, 1, 0, 47, 57, 0, 0, 0, 37, 0, 0)
7. QoS cut	236.0	(.99, .96, .96, .78, .14)	.778	(22, 35, 3, 0, 58, 45, 0, 33, 0, 4, 0, 0)
8. QoS cut	236.0	(.99, .95, .98, .86, .12)	.795	(28, 29, 5, 0, 49, 52, 0, 15, 0, 22, 0, 0)
9. QoS cut	236.0	(.99, .91, .97, .91, .12)	.796	(22, 35, 12, 0, 59, 35, 0, 23, 0, 14, 0, 0)
1. QoS cut	236.0	(.99, .94, .97, .87, .12)	.795	(26, 31, 8, 0, 52, 46, 0, 17, 0, 20, 0, 0)
11. QoS cut	236.2	(.99, .94, .98, .88, .12)	.798	(29, 28, 5, 0, 48, 52, 0, 11, 0, 27, 0, 0)
12. QoS cut	236.2	(.99, .92, .97, .90, .13)	.798	(27, 30, 8, 0, 51, 46, 0, 13, 0, 25, 0, 0)

How the algorithm behaves:

Example 1b, 500 hours, $\kappa = 0.20$, only overall QoS constraint.

iteration	obj.	QoS per call type	QoS	staffing vector
1. flow-cuts	233.0	(.99, .97, .60, .55, .25)	.685	(36, 36, 45, 0, 45, 1, 0, 37, 0, 0, 0, 0)
2. QoS cut	233.0	(.99, .96, .94, .50, .18)	.721	(36, 36, 16, 0, 45, 30, 0, 37, 0, 0, 0, 0)
3. QoS cut	233.0	—	—	(36, 36, 0, 0, 45, 46, 0, 19, 0, 18, 0, 0)
flow-cuts	233.0	(.99, .90, .98, .84, .08)	.773	(36, 36, 1, 0, 45, 45, 0, 19, 0, 18, 0, 0)
4. QoS cut	235.8	(.99, .97, .95, .75, .14)	.772	(36, 23, 0, 0, 45, 58, 0, 26, 0, 12, 0, 0)
5. QoS cut	235.8	(.99, .94, .97, .89, .10)	.793	(36, 22, 1, 0, 45, 59, 0, 9, 0, 28, 0, 0)
6. QoS cut	235.8	(.99, .86, .97, .93, .11)	.786	(34, 24, 1, 0, 47, 57, 0, 0, 0, 37, 0, 0)
7. QoS cut	236.0	(.99, .96, .96, .78, .14)	.778	(22, 35, 3, 0, 58, 45, 0, 33, 0, 4, 0, 0)
8. QoS cut	236.0	(.99, .95, .98, .86, .12)	.795	(28, 29, 5, 0, 49, 52, 0, 15, 0, 22, 0, 0)
9. QoS cut	236.0	(.99, .91, .97, .91, .12)	.796	(22, 35, 12, 0, 59, 35, 0, 23, 0, 14, 0, 0)
1. QoS cut	236.0	(.99, .94, .97, .87, .12)	.795	(26, 31, 8, 0, 52, 46, 0, 17, 0, 20, 0, 0)
11. QoS cut	236.2	(.99, .94, .98, .88, .12)	.798	(29, 28, 5, 0, 48, 52, 0, 11, 0, 27, 0, 0)
12. QoS cut	236.2	(.99, .92, .97, .90, .13)	.798	(27, 30, 8, 0, 51, 46, 0, 13, 0, 25, 0, 0)
13. QoS cut	236.2	(.99, .87, .97, .93, .14)	.795	(25, 32, 15, 0, 56, 34, 0, 15, 0, 23, 0, 0)

How the algorithm behaves:

Example 1b, 500 hours, $\kappa = 0.20$, only overall QoS constraint.

iteration	obj.	QoS per call type	QoS	staffing vector
1. flow-cuts	233.0	(.99, .97, .60, .55, .25)	.685	(36, 36, 45, 0, 45, 1, 0, 37, 0, 0, 0, 0)
2. QoS cut	233.0	(.99, .96, .94, .50, .18)	.721	(36, 36, 16, 0, 45, 30, 0, 37, 0, 0, 0, 0)
3. QoS cut	233.0	—	—	(36, 36, 0, 0, 45, 46, 0, 19, 0, 18, 0, 0)
flow-cuts	233.0	(.99, .90, .98, .84, .08)	.773	(36, 36, 1, 0, 45, 45, 0, 19, 0, 18, 0, 0)
4. QoS cut	235.8	(.99, .97, .95, .75, .14)	.772	(36, 23, 0, 0, 45, 58, 0, 26, 0, 12, 0, 0)
5. QoS cut	235.8	(.99, .94, .97, .89, .10)	.793	(36, 22, 1, 0, 45, 59, 0, 9, 0, 28, 0, 0)
6. QoS cut	235.8	(.99, .86, .97, .93, .11)	.786	(34, 24, 1, 0, 47, 57, 0, 0, 0, 37, 0, 0)
7. QoS cut	236.0	(.99, .96, .96, .78, .14)	.778	(22, 35, 3, 0, 58, 45, 0, 33, 0, 4, 0, 0)
8. QoS cut	236.0	(.99, .95, .98, .86, .12)	.795	(28, 29, 5, 0, 49, 52, 0, 15, 0, 22, 0, 0)
9. QoS cut	236.0	(.99, .91, .97, .91, .12)	.796	(22, 35, 12, 0, 59, 35, 0, 23, 0, 14, 0, 0)
1. QoS cut	236.0	(.99, .94, .97, .87, .12)	.795	(26, 31, 8, 0, 52, 46, 0, 17, 0, 20, 0, 0)
11. QoS cut	236.2	(.99, .94, .98, .88, .12)	.798	(29, 28, 5, 0, 48, 52, 0, 11, 0, 27, 0, 0)
12. QoS cut	236.2	(.99, .92, .97, .90, .13)	.798	(27, 30, 8, 0, 51, 46, 0, 13, 0, 25, 0, 0)
13. QoS cut	236.2	(.99, .87, .97, .93, .14)	.795	(25, 32, 15, 0, 56, 34, 0, 15, 0, 23, 0, 0)
14. QoS cut	**236.2**	(.99, .90, .97, .92, .16)	**.803**	(28, 29, 11, 0, 50, 44, 0, 8, 0, 30, 0, 0)

Same example, with QoS contraint of $\ell_k = 0.50$ per call type.

iteration	obj.	QoS per call type	QoS	staffing vector
1. flow-cuts	233.0	(.99, .97, .60, .55, .25)	.685	(36, 36, 45, 0, 45, 1, 0, 37, 0, 0, 0, 0)

Same example, with QoS contraint of $\ell_k = 0.50$ per call type.

iteration	obj.	QoS per call type	QoS	staffing vector
1. flow-cuts	233.0	(.99, .97, .60, .55, .25)	.685	(36, 36, 45, 0, 45, 1, 0, 37, 0, 0, 0, 0)
2. QoS type 5	234.2	—	—	(36, 36, 45, 1, 38, 1, 0, 43, 0, 0, 0, 0)
flow-cuts	235.4	(.99, .95, .62, .54, .41)	.711	(36, 36, 34, 0, 45, 0, 0, 48, 1, 0, 0, 0)

Same example, with QoS contraint of $\ell_k = 0.50$ per call type.

iteration	obj.	QoS per call type	QoS	staffing vector
1. flow-cuts	233.0	(.99, .97, .60, .55, .25)	.685	(36, 36, 45, 0, 45, 1, 0, 37, 0, 0, 0, 0)
2. QoS type 5	234.2	—	—	(36, 36, 45, 1, 38, 1, 0, 43, 0, 0, 0, 0)
flow-cuts	235.4	(.99, .95, .62, .54, .41)	.711	(36, 36, 34, 0, 45, 0, 0, 48, 1, 0, 0, 0)
3. QoS type 5	235.4	—	—	(36, 33, 40, 0, 43, 2, 0, 44, 2, 0, 0, 0)
flow-cuts	235.8	(.99, .96, .63, .51, .48)	.723	(31, 36, 41, 0, 45, 1, 0, 46, 0, 0, 0, 0)

Same example, with QoS contraint of $\ell_k = 0.50$ per call type.

iteration	obj.	QoS per call type	QoS	staffing vector
1. flow-cuts	233.0	(.99, .97, .60, .55, .25)	.685	(36, 36, 45, 0, 45, 1, 0, 37, 0, 0, 0, 0)
2. QoS type 5	234.2	—	—	(36, 36, 45, 1, 38, 1, 0, 43, 0, 0, 0, 0)
flow-cuts	235.4	(.99, .95, .62, .54, .41)	.711	(36, 36, 34, 0, 45, 0, 0, 48, 1, 0, 0, 0)
3. QoS type 5	235.4	—	—	(36, 33, 40, 0, 43, 2, 0, 44, 2, 0, 0, 0)
flow-cuts	235.8	(.99, .96, .63, .51, .48)	.723	(31, 36, 41, 0, 45, 1, 0, 46, 0, 0, 0, 0)
4. QoS type 5	235.8	(.99, .97, .48, .57, .47)	.706	(36, 30, 37, 0, 45, 7, 0, 45, 0, 0, 0, 0)

Same example, with QoS contraint of $\ell_k = 0.50$ per call type.

iteration	obj.	QoS per call type	QoS	staffing vector
1. flow-cuts	233.0	(.99, .97, .60, .55, .25)	.685	(36, 36, 45, 0, 45, 1, 0, 37, 0, 0, 0, 0)
2. QoS type 5	234.2	—	—	(36, 36, 45, 1, 38, 1, 0, 43, 0, 0, 0, 0)
flow-cuts	235.4	(.99, .95, .62, .54, .41)	.711	(36, 36, 34, 0, 45, 0, 0, 48, 1, 0, 0, 0)
3. QoS type 5	235.4	—	—	(36, 33, 40, 0, 43, 2, 0, 44, 2, 0, 0, 0)
flow-cuts	235.8	(.99, .96, .63, .51, .48)	.723	(31, 36, 41, 0, 45, 1, 0, 46, 0, 0, 0, 0)
4. QoS type 5	235.8	(.99, .97, .48, .57, .47)	.706	(36, 30, 37, 0, 45, 7, 0, 45, 0, 0, 0, 0)
5. QoS typ. 3,5	235.8	(.99, .97, .53, .53, .48)	.711	(33, 32, 41, 0, 45, 5, 0, 44, 0, 0, 0, 0)

Same example, with QoS contraint of $\ell_k = 0.50$ per call type.

iteration	obj.	QoS per call type	QoS	staffing vector
1. flow-cuts	233.0	(.99, .97, .60, .55, .25)	.685	(36, 36, 45, 0, 45, 1, 0, 37, 0, 0, 0, 0)
2. QoS type 5	234.2	—	—	(36, 36, 45, 1, 38, 1, 0, 43, 0, 0, 0, 0)
flow-cuts	235.4	(.99, .95, .62, .54, .41)	.711	(36, 36, 34, 0, 45, 0, 0, 48, 1, 0, 0, 0)
3. QoS type 5	235.4	—	—	(36, 33, 40, 0, 43, 2, 0, 44, 2, 0, 0, 0)
flow-cuts	235.8	(.99, .96, .63, .51, .48)	.723	(31, 36, 41, 0, 45, 1, 0, 46, 0, 0, 0, 0)
4. QoS type 5	235.8	(.99, .97, .48, .57, .47)	.706	(36, 30, 37, 0, 45, 7, 0, 45, 0, 0, 0, 0)
5. QoS typ. 3,5	235.8	(.99, .97, .53, .53, .48)	.711	(33, 32, 41, 0, 45, 5, 0, 44, 0, 0, 0, 0)
6. QoS type 5	236.0	—	—	(33, 34, 40, 0, 43, 3, 2, 45, 0, 0, 0, 0)
flow-cuts	236.0	(.99, .97, .55, .52, .48)	.710	(33, 33, 39, 0, 45, 4, 0, 46, 0, 0, 0, 0)

Same example, with QoS contraint of $\ell_k = 0.50$ per call type.

iteration	obj.	QoS per call type	QoS	staffing vector
1. flow-cuts	233.0	(.99, .97, .60, .55, .25)	.685	(36, 36, 45, 0, 45, 1, 0, 37, 0, 0, 0, 0)
2. QoS type 5	234.2	—	—	(36, 36, 45, 1, 38, 1, 0, 43, 0, 0, 0, 0)
flow-cuts	235.4	(.99, .95, .62, .54, .41)	.711	(36, 36, 34, 0, 45, 0, 0, 48, 1, 0, 0, 0)
3. QoS type 5	235.4	—	—	(36, 33, 40, 0, 43, 2, 0, 44, 2, 0, 0, 0)
flow-cuts	235.8	(.99, .96, .63, .51, .48)	.723	(31, 36, 41, 0, 45, 1, 0, 46, 0, 0, 0, 0)
4. QoS type 5	235.8	(.99, .97, .48, .57, .47)	.706	(36, 30, 37, 0, 45, 7, 0, 45, 0, 0, 0, 0)
5. QoS typ. 3,5	235.8	(.99, .97, .53, .53, .48)	.711	(33, 32, 41, 0, 45, 5, 0, 44, 0, 0, 0, 0)
6. QoS type 5	236.0	—	—	(33, 34, 40, 0, 43, 3, 2, 45, 0, 0, 0, 0)
flow-cuts	236.0	(.99, .97, .55, .52, .48)	.710	(33, 33, 39, 0, 45, 4, 0, 46, 0, 0, 0, 0)
7. QoS type 5	236.0	(.99, .97, .61, .51, .44)	.713	(25, 36, 52, 0, 45, 1, 0, 41, 0, 0, 0, 0)

Same example, with QoS contraint of $\ell_k = 0.50$ per call type.

iteration	obj.	QoS per call type	QoS	staffing vector
1. flow-cuts	233.0	(.99, .97, .60, .55, .25)	.685	(36, 36, 45, 0, 45, 1, 0, 37, 0, 0, 0, 0)
2. QoS type 5	234.2	—	—	(36, 36, 45, 1, 38, 1, 0, 43, 0, 0, 0, 0)
flow-cuts	235.4	(.99, .95, .62, .54, .41)	.711	(36, 36, 34, 0, 45, 0, 0, 48, 1, 0, 0, 0)
3. QoS type 5	235.4	—	—	(36, 33, 40, 0, 43, 2, 0, 44, 2, 0, 0, 0)
flow-cuts	235.8	(.99, .96, .63, .51, .48)	.723	(31, 36, 41, 0, 45, 1, 0, 46, 0, 0, 0, 0)
4. QoS type 5	235.8	(.99, .97, .48, .57, .47)	.706	(36, 30, 37, 0, 45, 7, 0, 45, 0, 0, 0, 0)
5. QoS typ. 3,5	235.8	(.99, .97, .53, .53, .48)	.711	(33, 32, 41, 0, 45, 5, 0, 44, 0, 0, 0, 0)
6. QoS type 5	236.0	—	—	(33, 34, 40, 0, 43, 3, 2, 45, 0, 0, 0, 0)
flow-cuts	236.0	(.99, .97, .55, .52, .48)	.710	(33, 33, 39, 0, 45, 4, 0, 46, 0, 0, 0, 0)
7. QoS type 5	236.0	(.99, .97, .61, .51, .44)	.713	(25, 36, 52, 0, 45, 1, 0, 41, 0, 0, 0, 0)
8. QoS type 5	236.2	(.99, .97, .55, .53, .50)	.718	(30, 32, 45, 0, 45, 5, 0, 43, 0, 0, 0, 0)

Same example, with QoS contraint of $\ell_k = 0.50$ per call type.

iteration	obj.	QoS per call type	QoS	staffing vector
1. flow-cuts	233.0	(.99, .97, .60, .55, .25)	.685	(36, 36, 45, 0, 45, 1, 0, 37, 0, 0, 0, 0)
2. QoS type 5	234.2	—	—	(36, 36, 45, 1, 38, 1, 0, 43, 0, 0, 0, 0)
flow-cuts	235.4	(.99, .95, .62, .54, .41)	.711	(36, 36, 34, 0, 45, 0, 0, 48, 1, 0, 0, 0)
3. QoS type 5	235.4	—	—	(36, 33, 40, 0, 43, 2, 0, 44, 2, 0, 0, 0)
flow-cuts	235.8	(.99, .96, .63, .51, .48)	.723	(31, 36, 41, 0, 45, 1, 0, 46, 0, 0, 0, 0)
4. QoS type 5	235.8	(.99, .97, .48, .57, .47)	.706	(36, 30, 37, 0, 45, 7, 0, 45, 0, 0, 0, 0)
5. QoS typ. 3,5	235.8	(.99, .97, .53, .53, .48)	.711	(33, 32, 41, 0, 45, 5, 0, 44, 0, 0, 0, 0)
6. QoS type 5	236.0	—	—	(33, 34, 40, 0, 43, 3, 2, 45, 0, 0, 0, 0)
flow-cuts	236.0	(.99, .97, .55, .52, .48)	.710	(33, 33, 39, 0, 45, 4, 0, 46, 0, 0, 0, 0)
7. QoS type 5	236.0	(.99, .97, .61, .51, .44)	.713	(25, 36, 52, 0, 45, 1, 0, 41, 0, 0, 0, 0)
8. QoS type 5	236.2	(.99, .97, .55, .53, .50)	.718	(30, 32, 45, 0, 45, 5, 0, 43, 0, 0, 0, 0)
9. QoS	239.8	(.99, .97, .88, .37, .57)	.753	(18, 36, 34, 0, 45, 14, 0, 53, 0, 0, 0, 0)

Same example, with QoS contraint of $\ell_k = 0.50$ per call type.

iteration	obj.	QoS per call type	QoS	staffing vector
1. flow-cuts	233.0	(.99, .97, .60, .55, .25)	.685	(36, 36, 45, 0, 45, 1, 0, 37, 0, 0, 0, 0)
2. QoS type 5	234.2	—	—	(36, 36, 45, 1, 38, 1, 0, 43, 0, 0, 0, 0)
flow-cuts	235.4	(.99, .95, .62, .54, .41)	.711	(36, 36, 34, 0, 45, 0, 0, 48, 1, 0, 0, 0)
3. QoS type 5	235.4	—	—	(36, 33, 40, 0, 43, 2, 0, 44, 2, 0, 0, 0)
flow-cuts	235.8	(.99, .96, .63, .51, .48)	.723	(31, 36, 41, 0, 45, 1, 0, 46, 0, 0, 0, 0)
4. QoS type 5	235.8	(.99, .97, .48, .57, .47)	.706	(36, 30, 37, 0, 45, 7, 0, 45, 0, 0, 0, 0)
5. QoS typ. 3,5	235.8	(.99, .97, .53, .53, .48)	.711	(33, 32, 41, 0, 45, 5, 0, 44, 0, 0, 0, 0)
6. QoS type 5	236.0	—	—	(33, 34, 40, 0, 43, 3, 2, 45, 0, 0, 0, 0)
flow-cuts	236.0	(.99, .97, .55, .52, .48)	.710	(33, 33, 39, 0, 45, 4, 0, 46, 0, 0, 0, 0)
7. QoS type 5	236.0	(.99, .97, .61, .51, .44)	.713	(25, 36, 52, 0, 45, 1, 0, 41, 0, 0, 0, 0)
8. QoS type 5	236.2	(.99, .97, .55, .53, .50)	.718	(30, 32, 45, 0, 45, 5, 0, 43, 0, 0, 0, 0)
9. QoS	239.8	(.99, .97, .88, .37, .57)	.753	(18, 36, 34, 0, 45, 14, 0, 53, 0, 0, 0, 0)
10. QoS type 4	240.0	(.99, .95, .77, .55, .53)	.763	(20, 36, 32, 0, 49, 7, 0, 56, 0, 0, 0, 0)

Same example, with QoS contraint of $\ell_k = 0.50$ per call type.

iteration	obj.	QoS per call type	QoS	staffing vector
1. flow-cuts	233.0	(.99, .97, .60, .55, .25)	.685	(36, 36, 45, 0, 45, 1, 0, 37, 0, 0, 0, 0)
2. QoS type 5	234.2	—	—	(36, 36, 45, 1, 38, 1, 0, 43, 0, 0, 0, 0)
flow-cuts	235.4	(.99, .95, .62, .54, .41)	.711	(36, 36, 34, 0, 45, 0, 0, 48, 1, 0, 0, 0)
3. QoS type 5	235.4	—	—	(36, 33, 40, 0, 43, 2, 0, 44, 2, 0, 0, 0)
flow-cuts	235.8	(.99, .96, .63, .51, .48)	.723	(31, 36, 41, 0, 45, 1, 0, 46, 0, 0, 0, 0)
4. QoS type 5	235.8	(.99, .97, .48, .57, .47)	.706	(36, 30, 37, 0, 45, 7, 0, 45, 0, 0, 0, 0)
5. QoS typ. 3,5	235.8	(.99, .97, .53, .53, .48)	.711	(33, 32, 41, 0, 45, 5, 0, 44, 0, 0, 0, 0)
6. QoS type 5	236.0	—	—	(33, 34, 40, 0, 43, 3, 2, 45, 0, 0, 0, 0)
flow-cuts	236.0	(.99, .97, .55, .52, .48)	.710	(33, 33, 39, 0, 45, 4, 0, 46, 0, 0, 0, 0)
7. QoS type 5	236.0	(.99, .97, .61, .51, .44)	.713	(25, 36, 52, 0, 45, 1, 0, 41, 0, 0, 0, 0)
8. QoS type 5	236.2	(.99, .97, .55, .53, .50)	.718	(30, 32, 45, 0, 45, 5, 0, 43, 0, 0, 0, 0)
9. QoS	239.8	(.99, .97, .88, .37, .57)	.753	(18, 36, 34, 0, 45, 14, 0, 53, 0, 0, 0, 0)
10. QoS type 4	240.0	(.99, .95, .77, .55, .53)	.763	(20, 36, 32, 0, 49, 7, 0, 56, 0, 0, 0, 0)
11. QoS	241.0	(.99, .97, .66, .68, .49)	.769	(22, 25, 41, 0, 48, 12, 0, 45, 0, 7, 0, 0)

Same example, with QoS contraint of $\ell_k = 0.50$ per call type.

iteration	obj.	QoS per call type	QoS	staffing vector
1. flow-cuts	233.0	(.99, .97, .60, .55, .25)	.685	(36, 36, 45, 0, 45, 1, 0, 37, 0, 0, 0, 0)
2. QoS type 5	234.2	—	—	(36, 36, 45, 1, 38, 1, 0, 43, 0, 0, 0, 0)
flow-cuts	235.4	(.99, .95, .62, .54, .41)	.711	(36, 36, 34, 0, 45, 0, 0, 48, 1, 0, 0, 0)
3. QoS type 5	235.4	—	—	(36, 33, 40, 0, 43, 2, 0, 44, 2, 0, 0, 0)
flow-cuts	235.8	(.99, .96, .63, .51, .48)	.723	(31, 36, 41, 0, 45, 1, 0, 46, 0, 0, 0, 0)
4. QoS type 5	235.8	(.99, .97, .48, .57, .47)	.706	(36, 30, 37, 0, 45, 7, 0, 45, 0, 0, 0, 0)
5. QoS typ. 3,5	235.8	(.99, .97, .53, .53, .48)	.711	(33, 32, 41, 0, 45, 5, 0, 44, 0, 0, 0, 0)
6. QoS type 5	236.0	—	—	(33, 34, 40, 0, 43, 3, 2, 45, 0, 0, 0, 0)
flow-cuts	236.0	(.99, .97, .55, .52, .48)	.710	(33, 33, 39, 0, 45, 4, 0, 46, 0, 0, 0, 0)
7. QoS type 5	236.0	(.99, .97, .61, .51, .44)	.713	(25, 36, 52, 0, 45, 1, 0, 41, 0, 0, 0, 0)
8. QoS type 5	236.2	(.99, .97, .55, .53, .50)	.718	(30, 32, 45, 0, 45, 5, 0, 43, 0, 0, 0, 0)
9. QoS	239.8	(.99, .97, .88, .37, .57)	.753	(18, 36, 34, 0, 45, 14, 0, 53, 0, 0, 0, 0)
10. QoS type 4	240.0	(.99, .95, .77, .55, .53)	.763	(20, 36, 32, 0, 49, 7, 0, 56, 0, 0, 0, 0)
11. QoS	241.0	(.99, .97, .66, .68, .49)	.769	(22, 25, 41, 0, 48, 12, 0, 45, 0, 7, 0, 0)
12. QoS type 5	241.2	(.99, .97, .49, .77, .48)	.758	(18, 25, 44, 0, 53, 11, 0, 46, 0, 3, 0, 0)

Same example, with QoS contraint of $\ell_k = 0.50$ per call type.

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2. QoS type 5	234.2	—	—	(36, 36, 45, 1, 38, 1, 0, 43, 0, 0, 0, 0)
flow-cuts	235.4	(.99, .95, .62, .54, .41)	.711	(36, 36, 34, 0, 45, 0, 0, 48, 1, 0, 0, 0)
3. QoS type 5	235.4	—	—	(36, 33, 40, 0, 43, 2, 0, 44, 2, 0, 0, 0)
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13. QoS typ. 3,5	241.2	(.99, .97, .54, .73, .50)	.761	(22, 24, 39, 0, 51, 12, 0, 47, 0, 5, 0, 0)

Same example, with QoS contraint of $\ell_k = 0.50$ per call type.

iteration	obj.	QoS per call type	QoS	staffing vector
1. flow-cuts	233.0	(.99, .97, .60, .55, .25)	.685	(36, 36, 45, 0, 45, 1, 0, 37, 0, 0, 0, 0)
2. QoS type 5	234.2	—	—	(36, 36, 45, 1, 38, 1, 0, 43, 0, 0, 0, 0)
flow-cuts	235.4	(.99, .95, .62, .54, .41)	.711	(36, 36, 34, 0, 45, 0, 0, 48, 1, 0, 0, 0)
3. QoS type 5	235.4	—	—	(36, 33, 40, 0, 43, 2, 0, 44, 2, 0, 0, 0)
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5. QoS typ. 3,5	235.8	(.99, .97, .53, .53, .48)	.711	(33, 32, 41, 0, 45, 5, 0, 44, 0, 0, 0, 0)
6. QoS type 5	236.0	—	—	(33, 34, 40, 0, 43, 3, 2, 45, 0, 0, 0, 0)
flow-cuts	236.0	(.99, .97, .55, .52, .48)	.710	(33, 33, 39, 0, 45, 4, 0, 46, 0, 0, 0, 0)
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12. QoS type 5	241.2	(.99, .97, .49, .77, .48)	.758	(18, 25, 44, 0, 53, 11, 0, 46, 0, 3, 0, 0)
13. QoS typ. 3,5	241.2	(.99, .97, .54, .73, .50)	.761	(22, 24, 39, 0, 51, 12, 0, 47, 0, 5, 0, 0)
14. QoS	241.2	(.99, .98, .82, .55, .53)	.779	(24, 24, 39, 0, 44, 22, 0, 43, 0, 5, 0, 0)

Same example, with QoS contraint of $\ell_k = 0.50$ per call type.

iteration	obj.	QoS per call type	QoS	staffing vector
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flow-cuts	235.4	(.99, .95, .62, .54, .41)	.711	(36, 36, 34, 0, 45, 0, 0, 48, 1, 0, 0, 0)
3. QoS type 5	235.4	—	—	(36, 33, 40, 0, 43, 2, 0, 44, 2, 0, 0, 0)
flow-cuts	235.8	(.99, .96, .63, .51, .48)	.723	(31, 36, 41, 0, 45, 1, 0, 46, 0, 0, 0, 0)
4. QoS type 5	235.8	(.99, .97, .48, .57, .47)	.706	(36, 30, 37, 0, 45, 7, 0, 45, 0, 0, 0, 0)
5. QoS typ. 3,5	235.8	(.99, .97, .53, .53, .48)	.711	(33, 32, 41, 0, 45, 5, 0, 44, 0, 0, 0, 0)
6. QoS type 5	236.0	—	—	(33, 34, 40, 0, 43, 3, 2, 45, 0, 0, 0, 0)
flow-cuts	236.0	(.99, .97, .55, .52, .48)	.710	(33, 33, 39, 0, 45, 4, 0, 46, 0, 0, 0, 0)
7. QoS type 5	236.0	(.99, .97, .61, .51, .44)	.713	(25, 36, 52, 0, 45, 1, 0, 41, 0, 0, 0, 0)
8. QoS type 5	236.2	(.99, .97, .55, .53, .50)	.718	(30, 32, 45, 0, 45, 5, 0, 43, 0, 0, 0, 0)
9. QoS	239.8	(.99, .97, .88, .37, .57)	.753	(18, 36, 34, 0, 45, 14, 0, 53, 0, 0, 0, 0)
10. QoS type 4	240.0	(.99, .95, .77, .55, .53)	.763	(20, 36, 32, 0, 49, 7, 0, 56, 0, 0, 0, 0)
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14. QoS	241.2	(.99, .98, .82, .55, .53)	.779	(24, 24, 39, 0, 44, 22, 0, 43, 0, 5, 0, 0)
15. QoS	241.8	(.99, .96, .85, .59, .52)	.787	(26, 38, 42, 0, 46, 3, 0, 46, 1, 2, 0, 0)

iteration	obj.	QoS per call type	QoS	staffing vector
⋮				

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iteration	obj.	QoS per call type	QoS	staffing vector
⋮				
15. QoS	241.8	(.99, .96, .85, .59, .52)	.787	(26, 38, 42, 0, 46, 3, 0, 46, 1, 2, 0, 0)
16. QoS	241.8	(.99, .96, .85, .59, .55)	.791	(26, 31, 42, 0, 43, 6, 0, 42, 5, 7, 0, 0)

iteration	obj.	QoS per call type	QoS	staffing vector
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15. QoS	241.8	(.99, .96, .85, .59, .52)	.787	(26, 38, 42, 0, 46, 3, 0, 46, 1, 2, 0, 0)
16. QoS	241.8	(.99, .96, .85, .59, .55)	.791	(26, 31, 42, 0, 43, 6, 0, 42, 5, 7, 0, 0)
17. QoS	242.0	(.99, .94, .87, .62, .51)	.791	(26, 33, 38, 0, 39, 2, 0, 47, 0, 16, 0, 0)

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17. QoS	242.0	(.99, .94, .87, .62, .51)	.791	(26, 33, 38, 0, 39, 2, 0, 47, 0, 16, 0, 0)
18. QoS	242.0	(.99, .96, .82, .62, .52)	.789	(23, 30, 38, 0, 44, 9, 0, 47, 0, 10, 0, 0)

iteration	obj.	QoS per call type	QoS	staffing vector
⋮				
15. QoS	241.8	(.99, .96, .85, .59, .52)	.787	(26, 38, 42, 0, 46, 3, 0, 46, 1, 2, 0, 0)
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18. QoS	242.0	(.99, .96, .82, .62, .52)	.789	(23, 30, 38, 0, 44, 9, 0, 47, 0, 10, 0, 0)
19. QoS	242.0	(.99, .96, .83, .61, .52)	.789	(31, 32, 39, 0, 41, 5, 0, 44, 0, 11, 0, 0)

iteration	obj.	QoS per call type	QoS	staffing vector
⋮				
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17. QoS	242.0	(.99, .94, .87, .62, .51)	.791	(26, 33, 38, 0, 39, 2, 0, 47, 0, 16, 0, 0)
18. QoS	242.0	(.99, .96, .82, .62, .52)	.789	(23, 30, 38, 0, 44, 9, 0, 47, 0, 10, 0, 0)
19. QoS	242.0	(.99, .96, .83, .61, .52)	.789	(31, 32, 39, 0, 41, 5, 0, 44, 0, 11, 0, 0)
20. QoS	242.2	(.99, .95, .87, .61, .53)	.796	(27, 34, 36, 0, 41, 4, 0, 48, 0, 12, 0, 0)

iteration	obj.	QoS per call type	QoS	staffing vector
⋮				
15. QoS	241.8	(.99, .96, .85, .59, .52)	.787	(26, 38, 42, 0, 46, 3, 0, 46, 1, 2, 0, 0)
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18. QoS	242.0	(.99, .96, .82, .62, .52)	.789	(23, 30, 38, 0, 44, 9, 0, 47, 0, 10, 0, 0)
19. QoS	242.0	(.99, .96, .83, .61, .52)	.789	(31, 32, 39, 0, 41, 5, 0, 44, 0, 11, 0, 0)
20. QoS	242.2	(.99, .95, .87, .61, .53)	.796	(27, 34, 36, 0, 41, 4, 0, 48, 0, 12, 0, 0)
21. QoS	242.4	(.99, .95, .88, .61, .54)	.798	(26, 33, 37, 0, 41, 6, 0, 47, 0, 12, 0, 0)

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17. QoS	242.0	(.99, .94, .87, .62, .51)	.791	(26, 33, 38, 0, 39, 2, 0, 47, 0, 16, 0, 0)
18. QoS	242.0	(.99, .96, .82, .62, .52)	.789	(23, 30, 38, 0, 44, 9, 0, 47, 0, 10, 0, 0)
19. QoS	242.0	(.99, .96, .83, .61, .52)	.789	(31, 32, 39, 0, 41, 5, 0, 44, 0, 11, 0, 0)
20. QoS	242.2	(.99, .95, .87, .61, .53)	.796	(27, 34, 36, 0, 41, 4, 0, 48, 0, 12, 0, 0)
21. QoS	242.4	(.99, .95, .88, .61, .54)	.798	(26, 33, 37, 0, 41, 6, 0, 47, 0, 12, 0, 0)
22. QoS	242.6	(.99, .95, .84, .63, .53)	.794	(27, 32, 39, 0, 41, 3, 0, 46, 0, 14, 0, 0)

iteration	obj.	QoS per call type	QoS	staffing vector
⋮				
15. QoS	241.8	(.99, .96, .85, .59, .52)	.787	(26, 38, 42, 0, 46, 3, 0, 46, 1, 2, 0, 0)
16. QoS	241.8	(.99, .96, .85, .59, .55)	.791	(26, 31, 42, 0, 43, 6, 0, 42, 5, 7, 0, 0)
17. QoS	242.0	(.99, .94, .87, .62, .51)	.791	(26, 33, 38, 0, 39, 2, 0, 47, 0, 16, 0, 0)
18. QoS	242.0	(.99, .96, .82, .62, .52)	.789	(23, 30, 38, 0, 44, 9, 0, 47, 0, 10, 0, 0)
19. QoS	242.0	(.99, .96, .83, .61, .52)	.789	(31, 32, 39, 0, 41, 5, 0, 44, 0, 11, 0, 0)
20. QoS	242.2	(.99, .95, .87, .61, .53)	.796	(27, 34, 36, 0, 41, 4, 0, 48, 0, 12, 0, 0)
21. QoS	242.4	(.99, .95, .88, .61, .54)	.798	(26, 33, 37, 0, 41, 6, 0, 47, 0, 12, 0, 0)
22. QoS	242.6	(.99, .95, .84, .63, .53)	.794	(27, 32, 39, 0, 41, 3, 0, 46, 0, 14, 0, 0)
23. QoS	**242.6**	(.99, .96, .81, .67, .53)	**.801**	(25, 34, 41, 0, 45, 4, 0, 46, 0, 8, 0, 0)

Example 2a. 20 call types. 15 agent types.
Arrival rates λ_k vary between 125 and 260 (total $= 4080$); $\mu_k = 12$ for all k.
No abandonment; $\ell = .80$; simulate 500 hours.

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No abandonment; $\ell = .80$; simulate 500 hours.

κ	obj.	QoS	QoS KT	cuts	CPU (sec)
QoS per call type					
$\ell_k = 0$					
.20	627.0	$.802 \pm .002$	.084	6	1239
(1.0, .99, .76, .91, .99, .99, .98, .80, .00, .89, .98, .98, .97, .94, .92, .93, .91, .51, .21, .00)					
$\ell_k = 0.50$					
.20	667.8	$.841 \pm .006$	.689	50	6312
(.99, .99, .98, .66, .97, .99, .97, .98, .54, .89, .96, .97, .94, .99, .51, .83, .53, .87, .76, .72)					

Example 2b. Same, except: abandonment with prob. 0.01, then at rate 10.0.

κ	obj.	QoS	cuts	CPU (sec)	
QoS per call type					
$\ell_k = 0$					
.20	551.0	$.800 \pm .002$	1	107	
(.99, .99, .60, .33, .99, .98, .88, .77, .34, .84, .79, .99, .98, .96, .97, .68, .99, .32, .79, .70)					
$\ell_k = 0.50$					
.20	553.8	$.801 \pm .002$	6	305	
(.99, .98, .68, .50, .98, .99, .57, .80, .53, .77, .76, .99, .99, .87, .99, .66, .98, .74, .50, .81)					

Example 2c. Same, except: Abandonment with prob. 0.01, then at rate 10.0. Arrival rates vary between 225 and 360 (total = 5750) (more traffic).

κ	obj.	QoS	cuts	CPU (sec)	
QoS per call type					
$\ell_k = 0$					
.20	859.6	$.800 \pm .002$	11	2186	
(.99, .99, .77, .27, .99, .99, .92, .94, .01, .66, .88, .99, .99, .98, .98, .51, .98, .88, .61, .57)					
$\ell_k = 0.50$					
.20	916.2	$.854 \pm .003$	15	1197	
(.99, .99, .98, .95, .99, .99, .95, .93, .87, .52, .99, .94, .89, .93, .59, .99, .84, .76, .50, .51)					

A Blend System

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Problem: want to select a collection of shifts for agents to minimize costs subject to a QoS constraint on inbound calls.

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Perhaps another QoS constraint on the expected volume of successful outbound calls, or they can be incorporated in the cost function.

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An agent can be inbound-only or blend.

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Can use detailed simulation or simplified approximations.

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M1. All blend agents, $\mu_1 = \mu_2$, one dial at a time. Birth-death.

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M1. All blend agents, $\mu_1 = \mu_2$, one dial at a time. Birth-death.
M3. Two agent types, $\mu_1 \neq \mu_2$, several dials at a time.

Problem: want to select a collection of shifts for agents to minimize costs subject to a QoS constraint on inbound calls.

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We have developed steady-state CTMC models of varying complexities (Deslauriers, L'Ecuyer, Pichitlamken, Ingolfsson, Avramidis 2004):

M1. All blend agents, $\mu_1 = \mu_2$, one dial at a time. Birth-death.
M3. Two agent types, $\mu_1 \neq \mu_2$, several dials at a time.
Etc.

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exponential connection delay; successful with probability κ.

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Finite queue length c.

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QoS: (80, 20) rule.

Number of admissible shifts: 110.
Each shift: 7 working hours + 1 hour lunch (movable).

Period	Start	λ_i	κ_i	$1/\nu_i$	$1/\mu_{1,i}$	$1/\mu_{2,i}$	agents	
							simul.	M1
1	8.0	26.1	0	400	595.6	440.2	8	9
2	8.5	38.3	0	400	595.6	440.2	13	16
3	9.0	50.9	0	400	595.6	440.2	20	22
4	9.5	58.0	0	700	595.6	440.2	24	25
5	10.0	61.4	0	700	595.6	440.2	24	25
6	10.5	62.2	0	600	595.6	440.2	24	25
7	11.0	62.6	0	600	595.6	440.2	27	25
8	11.5	60.5	0	600	595.6	440.2	27	29
9	12.0	56.8	0	600	575.1	440.2	26	26
10	12.5	56.7	0	600	575.1	440.2	23	25
11	13.0	59.0	0.30	500	575.1	440.2	24	23
12	13.5	57.3	0.33	500	575.1	440.2	27	32
13	14.0	56.5	0.27	500	541.5	440.2	26	23
14	14.5	55.4	0.27	500	518.2	440.2	31	23
15	15.0	56.5	0.28	500	509.2	440.2	27	31
16	15.5	59.0	0.29	500	506.0	440.2	27	31
17	16.0	59.9	0.29	500	512.8	440.2	28	28
18	16.5	54.7	0.30	500	505.7	440.2	23	21
19	17.0	43.5	0.33	500	505.2	440.2	16	15
20	17.5	40.0	0.37	500	491.0	440.2	12	12
21	18.0	33.3	0.40	500	494.9	440.2	12	12
22	18.5	29.7	0.38	500	471.9	440.2	12	12
23	19.0	25.0	0.41	500	468.4	440.2	9	12
24	19.5	22.5	0.41	100	462.6	440.2	9	8
25	20.0	17.9	0.41	50	460.4	440.2	5	8

Optim. with	CPU (sec)	num. shifts	QoS with M1	QoS with simul.
M1	18	37	**.803**	.815
simulation	2349	36	.785	**.803**

Ongoing and Future Work in Our Group

- Develop a Java library for simulation of call centers.
- Integrate with optimization algorithms (CPLEX-based IP, perhaps Lagrangian relaxation, neighborhood search, etc.) for staffing, scheduling, rostering, routing, ...
- Integrate with approximation formulas and CTMC models.
- Better models for inbound traffic and other aspects.
- Currently, our work is strongly driven by the interests of Bell Canada.