Capacity Management in Call Centers
Basic Models and Links to Current Research

from a review article authored with Ger Koole and Avishai Mandelbaum
Outline:

- Tutorial
  - background on how calls are handled
  - basic model of call-center operations

- Research overview
  - system primitives
  - basic model
  - routing and networking
  - further directions for research
Tutorial, background...

Typical call-center environments:

Top figure from Larréché et al. (1997).
Representative infrastructure for a single location:
Queueing model associated with a single location:

Tutorial, background...

Queueing model associated with a single location:
Tutorial, basic model...

Traditional model used in capacity management:

- Work arriving over time
- Short-term capacity requirements
- Medium-term shift scheduling and rostering
- Long-term hiring and training
Tutorial, basic model...

Number of calls arriving over several time horizons:

...each month of the year

...each day of the month

...each hour of the day

...each minute of the hour
Tutorial, basic model...

Work modelled as arriving in discrete time buckets:

\[ R_i = \frac{\lambda_i}{\mu} = \text{work arriving during } i \]
Tutorial, basic model...

Waiting-time standards drive capacity requirements:

- ASA = average speed of answer
  \[ E[\text{wait}] \]

- TSF = telephone service factor (or service level)
  \[ P\{\text{wait} \leq T\} \]

- Fraction of calls abandoned before served
  \[ P\{\text{abandon}\} \]

- Determining capacity requirements for interval \( i \):
  \[ N_i = \min\{N \mid E[\text{wait}] \leq ASA^*\} \]

\[ \text{e.g.} \]
A report with 1/2-hour intervals:

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<tr>
<th>Time</th>
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Tutorial, basic model...

Performance estimate uses $M/M/N/\infty$ model:

- no blocking, abandonment, or retrials
- fixed arrival and service rates
- exponential interarrival and service times
- measures of stationary performance
Tutorial, basic model...

Estimates require only $\lambda_i, \mu, N$:

- $P\{\text{wait} > 0\}$
  \[
  = 1 - \frac{\sum_{m=0}^{N-1} R_i^m / m!}{\sum_{m=0}^{N-1} R_i^m / m! + (R_i^N / N!) / (1 - R_i/N)}
  \]
  (Erlang-C formula)

- $P\{\text{wait} \leq T\}$
  \[
  = 1 - P\{\text{wait} > 0\} \cdot P\{\text{wait} > T|\text{wait} > 0\}
  = 1 - P\{\text{wait} > 0\} \cdot e^{-(N\mu - \lambda_i)}
  \]

- $E[\text{wait}]$
  \[
  = P\{\text{wait} > 0\} \cdot E[\text{wait}|\text{wait} > 0]
  = P\{\text{wait} > 0\} \cdot (N\mu - \lambda_i)^{-1}
  \]
Example with 30-second ASA:

$N_i = \text{number of CSRs working in } i$
Determining staffing requirements for a day or week:

- **Data**
  
  \[ N_i = \text{num. CSRs required for interval } i \]
  
  \[ c_j = \text{cost of putting a CSR on schedule } j \]
  
  \[ a_{ij} = \begin{cases} 
  1 & \text{if sched. } j \text{ take calls during int. } i, \\
  0 & \text{otherwise.} 
\end{cases} \]

- **Decision variables**
  
  \[ x_j = \text{num. CSRs to work on schedule } j \]

- **Math program to choose schedules**
  
  \[ \min \{ c'x \mid Ax \geq N; x \geq 0; x \text{ integer} \} \]
Tutorial, basic model...

Example with 4-hour shifts on the 1/2-hour:

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Excess capacity resulting from the scheduling IP:
More on the scheduling process:

- Scheduling IP recommends schedules

- Rostering assigns CSRs to schedules (shift-bidding)

- Scheduling IP also recommends a number of CSRs, $1'x$

- Target headcount $1'x$ typically inflated to accommodate “shrinkage”

- Target headcount $1'x$ can be used in a longer-term hiring problem
Tutorial, basic model...

Longer-term hiring and training:

- $n_t = 1^t x_t$
  - target headcount for scheduling period $t$

- $\tau$
  - lead time required to hire and train a new CSR

- Then in scheduling period $t$ hire

$$n_{t+\tau} - \sum_{j=t}^{t+\tau-1} y_j (1 - \beta)^{\tau - (j-t)}$$

where

- $n_{t+\tau} = \text{target headcount for hiring today}$
- $y_j = \text{CSRs that start working in period } j$
- $\beta = 1$-period attrition rate
Tutorial, basic model...

ACD reporting system drives capacity planning:

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<td>222.0</td>
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<td>82.8%</td>
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</tr>
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</tr>
<tr>
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<td>302</td>
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<td>314</td>
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</tr>
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<tr>
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<td>307</td>
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<td>81.8%</td>
<td>160.0</td>
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<td>2</td>
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<td>135.0</td>
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<tr>
<td>17:30</td>
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<td>103.5</td>
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<td>18:00</td>
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<td>14</td>
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<td>99.1%</td>
<td>5.8</td>
<td>1.4</td>
<td>416.2%</td>
</tr>
</tbody>
</table>
The arrival process:

- **Time-inhomogeneous process**
  - Andrews and Cunningham (1995)
  - Massey et al. (1996)
  - Brown et al. (2002)
  - Avramidis et al. (2004)
  - Huang and Shen (2004)

- **Uncertain rates**
  - Whitt (1999)
  - Jongbloed and Koole (2001)
  - Brown et al. (2002)
  - Avramidis et al. (2004)

- **Correlation among rates**
  - Andrews and Cunningham (1995)
  - Whitt (1999)
  - Brown et al. (2002)
  - Avramidis et al. (2004)
Service times:

- Exponential distribution
  - Kort (1983)
  - Harris et al. (1987)

- Gamma distribution
  - Chlebus (1997)

- Lognormal distribution
  - Bolotin (1994)
  - Mandelbaum et al. (2001)
  - Brown et al. (2002)
Abandonment:

- Earlier work on impatience and abandonment
  - Palm (1943)
  - Kort (1983)
  - Roberts (1979)

- Abandonment behavior in a call center
  - Mandelbaum et al. (2001)
  - Zohar et al. (2002)
  - Brown et al. (2002)
Research, basic model...

System performance and limits:

- Many-server limits for $M/M/N$ model
  - Halfin and Whitt (1982)
  - Borst et al. (2004)

- Systems with abandonment
  - Baccelli and Hebuterne (1981)
  - Brandt and Brandt (1999a,b)
  - Garnett et al. (2002)
  - Whitt (2004a,b,c,d)

- Also blocking, retrials, dependent $\lambda$...
  - Jagerman (1974)
  - Massey and Wallace (2002)
  - Aguir et al. (2004)
Research, basic model...

Staffing for a more complex arrival process:

- **Time-varying rates – numerical methods**
  - Yoo (1996)
  - Ingolfsson et al. (2002)

- **Time-varying rates – approximate methods**
  - fluids – Mandelbaum et al. (1995...2000)
  - $M_t/G/\infty$ systems
    - Eick et al. (1993a,b)
    - Jennings et al. (1996)
    - Massey and Whitt (1997)
    - Whitt (1999)

- **Uncertain arrival rates (and absenteeism)**
  - Ross (2001)
  - Chen and Henderson (2001)
  - Jongbloed and Koole (2001)
  - Whitt (2004e)
  - Harrison and Zeevi (2005)
Higher-level problems:

- Solutions to scheduling and rostering problems
  - break placement: Segal (1974)
  - column restriction
    Henderson and Berry (1976)
  - alternative formulations
    Aykin (1996), Brusco and Jacobs (2000)
  - rostering: Thompson (1997)

- Joint staffing and scheduling
  - Yoo (1996), Ingolfsson at al. (2000,2002)
  - Atlason et al. (2002)
  - Koole and van der Sluis (2003)

- Long-term hiring and training
  - Akşin (2002)
  - Gans and Zhou (2002)
Skills-based routing:

- The general routing problem

  ![Diagram of types of calls and feasible routings between pools of CSRs]

- Special routing structures

  ![Diagram of special routing structures labeled "I" to "M"]

Research, routing and networking...
Non-scaled analysis:

- **Exact analysis of special designs**
  - Bhulai and Koole (2000)

- **Heuristic policies for special designs**
  - Brandt and Brandt (1997)
  - Stanford and Grassmann (2000)
  - Stolletz and Helber (2004)
  - Chevalier et al. (2004)

- **Heuristic policies for more general designs**
  - Koole and Talim (2000)
  - Borst and Seri (2000)
Asymptotic analysis:

• “ED” regime
  – thresholds, N-design – Bell and Williams (2001)
  – $Gc\mu$ rules – Mandelbaum and Stolyar (2003)

• “QED” regime
  – Armony and Maglaras (2004a,b)
  – Atar et al. (2004)

• Other scalings
  – Harrison and Zeevi (2005)
Research, routing and networking...

Networking:

- Network structures

  - Some recent telecom research
    - Borst et al. (1996)
    - Kogan et al. (1997)
    - Servi and Humair (1999)

- SBR methods may also apply
On the horizon...

Examples of further directions for research:

• A broader view of the service process

• Better understanding of customer and CSR behavior
  – Carmon and Kahneman (2002)

• Multiple levels of equilibria
  – Armony and Maglaras (2004a, b)

• Customer relationship management by the numbers
  – Ariely et al. (2002)