Queueing systems with many servers: Control theory and heavy traffic asymptotics

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Abstract

A control theoretic approach to queues in heavy traffic analyzes an optimal control problem for a diffusion model so as to construct from it asymptotically optimal controls for the queueing model. Models for call centers, consisting of several *classes* of customers and many servers of different *types* working in parallel, introduce some challenges to both aspects of the approach.

Referring to class-type pairs as *activities*, an underlying fluid-level allocation problem identifies the fraction of the number of servers allocated to each activity (out of all servers of a given type). Activities for which the allocation fraction is positive are said to be *basic*. Under a condition known as *complete resource pooling*, the graph formed by all basic activities is a tree (by results of Harrison and López (1999)).

We will describe some recent results that show that the nonbasic activities, unseen in the fluid model, have a dramatic effect on the diffusion (and, consequently, queueing) model. In fact, the diffusion control problem turns out to accommodate two modes of control. The first corresponds to migration of customers between different service stations *along the tree*; this amounts to control that enters in the drift coefficient. The second corresponds to motion in circles *through nonbasic activities and across the tree*, and enters as singular terms, in the form of controllable increasing processes in certain directions of the state space. Depending on these directions, one of two things may happen: Either the problem degenerates so much as to make it

possible to maintain empty queues at all times; or a genuine singular control problem arises.

We will also describe results and open problems regarding the treatment of the diffusion model via the corresponding HJB equation and its use in obtaining asymptotically optimal schemes.

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