

## Measuring ultrametricity and clustering properties of disordered systems

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We present results [1] from Monte Carlo simulations to test for ultrametricity (UM) and clustering properties in spin-glass models. For this purpose, we introduce a different normalization of an UM measure, which allows to distinguish real UM from the spurious UM of the paramagnetic phase, in contrast to previous approaches. Furthermore, we extend a hierarchical clustering approach [2], resulting in displaying the hierarchical organization of a set of configurations via a dendrogram and a grey scale picture of the matrix  $d$  of hamming distances. The extension allows to count the number of top-level clusters of the configuration space in a well defined way. These approaches are applicable to a wide range of models in disordered systems, like spin glasses [3] or combinatorial optimization problems [4], which are well known for their complex configuration landscapes.

By using a one-dimensional Ising spin glass with random power-law interactions [5], where the universality class of the model can be tuned by changing the power-law exponent, we find signatures of UM behavior both in the mean-field and non-mean-field universality classes for large linear system sizes. Furthermore, we confirm the existence of nontrivial connected components in phase space via a clustering analysis of configurations. Also selected results from applications of these methods to other models like the vertex-cover problem, the number-partitioning problem and the satisfiability problem are given.

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[5] G. Kotliar, P. W. Anderson, and D. L. Stein, *Phys. Rev. B* **27** (1983) R602.