Workshop on Spectrum and Dynamics
Atelier sur la dynamique et théorie spectrale
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Minicourse: Mixing and waves: On the long time behaviour of wave propagation on manifolds of negative curvature.

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Abstract

It is well known that wave propagation on a Riemannian manifold is in the limit of short wavelengths driven by the geodesic flow, so one might ask how the dynamical properties of the geodesic flows are reflected in the propagation of waves. The geodesic flow on a compact manifold of negative curvature is the classical example of an Anosov system, in particular it is exponentially mixing and satisfies a central limit theorem. We will see how this leads to similar properties of propagated waves for large times.

A crucial problem in this analysis is the presence of two asymptotic limits, small wavelengths and large times, and the main difficulties lie in understanding their relation and the different time scales which emerge.

In this mini-course we will discuss in detail the timescales in this problem and how to extend the validity of the present techniques for approximating wave propagation in the short-wavelengths limit to larger times.

Lecture 1: Everything you always wanted to know about the Ehrenfest time - *But Were Afraid to Ask.*

I will recall a few facts about semiclassical approximations and pseudodifferential operators and show how the Ehrenfest time appears as a limit of our current techniques. I will then discuss several aspects of it and try to elucidate its meaning in relation to the classical dynamics. We

will then discuss how the Ehrenfest time limits our understanding of several problems in wave-propagation and spectral asymptotics, in particular equidistribution of waves and eigenfunctions, and the remainder term in Weyl's law.

Lecture 2: Beyond the Ehrenfest time.

We will discuss recent progress on the validity of semiclassical approximations beyond the Ehrenfest time on manifolds of constant negative curvature.

In the second week I plan to give two more informal lectures:

Lecture 3: Dispersion on the hyperbolic plane.

I will give a bit more details on the proofs of the results described in Lecture 2. We have seen that in order to go beyond the Ehrenfest time we need precise estimates on dispersion. We will give some more details on these estimates and discuss possible improvements which would even further extend the times up to which semiclassical approximations work.

Lecture 4: Mixing of waves beyond the Ehrenfest time.

I will discuss work in progress on how one can combine the long time semiclassical approximations with exponential mixing to obtain equidistribution and maybe other statistical properties of propagated waves for large times.