Workshop on Spectrum and Dynamics
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Minicourse: Entropy of chaotic eigenmodes

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Abstract

Our aim is to understand the (phase) space structure of the eigenstates of quantized Anosov systems, in the semiclassical (or high-energy) limit. The models we have considered are

- high-energy eigenmodes of the Laplacian on compact Riemannian manifolds of negative curvature
- eigenstates of quantized Anosov maps on the 2-torus.

The asymptotic phase space distribution is described by the semiclassical (limit) measures associated with sequences of eigenmodes. These probability measures are invariant probability measures w.r.to the classical dynamics. One way to characterize the localization of an invariant measure is to compute its Kolmogorov–Sinai entropy (the more concentrated the measure, the smaller its entropy). In this work we show that the entropy of semiclassical measures is bounded from below by (roughly speaking) half the maximal entropy. To summarize, semiclassical chaotic eigenstates are "at least half-delocalized". This result should be compared with the Schnirelman's theorem, stating that almost all eigenstates are "fully delocalized", and that this may be the case for all Laplacian eigenmodes.

Among standard semiclassical tools, our proof uses new "hyperbolic dispersion estimate" on the quantum propagator, which are valid beyond the "Ehrenfest time", and an "entropic uncertainty principle" adapted to our setting.

The first lecture will present the results and sketch the proof in the case of the Laplacian eigenstates. The second lecture will provide more details on these two ingredients. The third lecture will present the case of two quantum maps, where explicit half-delocalized eigenstates can be exhibited.

Joint work with Nalini Anantharaman and Herbert Koch.