Workshop on Spectrum and Dynamics
Atelier sur la dynamique et théorie spectrale
07–11 April/*Avril*, 2008

The transfer operator approach for Selberg's zeta function for Hecke triangle groups

Dieter MAYER
Institut für Theoretische Physik
Technische Universität Clausthal
Arnold-Sommerfeld-Strasse 6
38678 Clausthal-Zellerfeld
GERMANY

dieter.mayer@tu-clausthal.de

Abstract

We extend the transfer operator approach to Selberg's zeta function for the Hecke triangle groups G_q. These are cofinite Fuchsian groups acting on the upper half plane $\mathbb{H}$ with generators $\langle S, T_{\lambda_q}\rangle$ with $Sz = \frac{-1}{z}$ and $T_{\lambda_q} z = z + \lambda_q$ where $\lambda_q = 2\cos\frac{\pi}{q}$. We construct a symbolic dynamics for the geodesic flow on the corresponding Hecke surfaces $M_q = G_q \setminus \mathbb{H}$ by using the Hurwitz-Nakada continued fraction expansion determined by the map $f_q : I_q \to I_q$ of the interval $I_q = [-\frac{\lambda_q}{2}, \frac{\lambda_q}{2}]$ defined by $f_q(x) = -\frac{1}{x} - \lfloor \frac{-1}{x\lambda_q} + \frac{1}{2} \rfloor \lambda_q$. It turns out that the natural extension of f_q can be used as a Poincare map for the flow and hence its periodic points are directly related to the closed orbits of the flow. This allows us to define a transfer operator whose Fredholm determinant is closely related to the Selberg zeta function for these Hecke triangle groups.