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On Nodal domains and spectral minimal partitions

(After B. Helffer, T. Hoffmann-Ostenhof, S. Terracini, G. Vial,
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Abstract

Given an open set Ω and a partition of Ω by k open sets ω_j , we can consider the quantity $\max_j \lambda(\omega_j)$ where $\lambda(\omega_j)$ is the ground state energy of the Dirichlet realization of the Laplacian in ω_j . If we denote by $\mathfrak{L}_k(\Omega)$ the infimum over all the k -partitions of $\max_j \lambda(\omega_j)$, a minimal k -partition is then a partition which realizes the infimum. Although the analysis is rather standard when $k = 2$ (we find the nodal domains of a second eigenfunction), the analysis of higher k 's becomes non trivial and quite interesting. In this talk, we would like to discuss the properties of minimal spectral partitions, illustrate the difficulties by considering simple cases like the disc or the square ($k = 3$) and will also exhibit the possible role of the hexagone in the asymptotic behavior as $k \rightarrow +\infty$ of $\mathfrak{L}_k(\Omega)$.

This work has started in collaboration with T. Hoffmann-Ostenhof and has been continued (published or in progress) with the coauthors mentioned above.