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Invariant Tori of full dimension for a nonlinear Schrödinger equation

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Abstract

In this talk, we consider the one-dimensional nonlinear Schrödinger equation

$$\mathrm{i}u_t - u_{xx} + mu + f(|u|^2)u = 0$$

with periodic boundary conditions or Dirichlet boundary conditions, where f is a real analytic function in some neighborhood of the origin satisfying $f(0) = 0$, $f'(0) \neq 0$. We prove that for each given constant potential m, the equation admits a Whitney smooth family of small–amplitude, time almost–periodic solutions with all frequencies. The proof is based on a Birkhoff normal form reduction and an improved version of the KAM theorem. Thus, we give an affirmative answer to an open problem stated in [Pöschel, Erg. Th. Dynam. Syst. **22** (2002), 1537–1549; Bourgain, J. Funct. Anal. **229** (2005), 62–94].