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Birkhoff normal form and almost global existence for Hamiltonian PDEs

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Abstract

The method of Birkhoff normal form was introduced in the first years of the 20th century, but only recently some generalizations to PDEs have been obtained. The main issue of this method is a complete description of the qualitative features of solutions of small amplitude. In particular one gets long times estimates for the Sobolev norms of the solutions. This allows to deduce lower estimates on the time of existence of solutions.

In the present lecture I will give a review of the classical Birkhoff normal form theory and present some results for nonlinear PDEs taking as a model probelm the wave equation on the d-dimensional sphere. I will also discuss some open problems.