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Global bifurcation structure of a Ginzburg–Landau model in a loop

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Abstract

We consider the following Ginzburg–Landau energy functional defined in $H^1(\mathbb{R}/2\pi;\mathbb{C})$, which is a simple model with an applied magnetic field in a superconducting loop with the uniform cross section,

$$E(\psi) := \int_0^{2\pi} |D_h\psi|^2 + \frac{\lambda}{2}(1-|\psi|^2)^2 dx, \quad D_h := d/dx - ih(x), \tag{1}$$

where $\psi(x)$ is a complex-valued order parameter, λ is a positive parameter and $h(x)$ is a real-valued periodic function. We investigate solutions to the Euler–Lagrange equation of (1),

$$\begin{cases} D_h^2\psi + \lambda(1-|\psi|^2)\psi = 0, & x \in \mathbb{R}, \\ \psi(x+2\pi) = \psi(x), & x \in \mathbb{R}. \end{cases} \tag{2}$$

Besides the trivial solution $\psi = 0$, the equation (2) allows 3 types of solutions:

(i) solutions with constant amplitude,

(ii) solutions with modulating amplitude, and

(iii) solutions with zeros.

As for the former two types of solutions, we can define the winding number of the solutions around the origin in the complex plane. Setting $\mu = \frac{1}{2\pi}\int_0^{2\pi} h(x)\,dx$, we solve all the solutions to (2) and reveal the bifurcation structure in the parameter space (μ, λ). In particular we show how the winding number of the amplitude-modulating solution changes as μ varies.

The main results are due to the work by *Kosugi–Morita–Yotsutani*.