Relaxed energies for $H^{1/2}$-maps with values into the circle

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Abstract

We consider, for maps defined on the plane with values into the circle, an energy E associated to a measurable matrix field on the half space $\mathbb{R}^3_+$ and related to a seminorm equivalent to the $H^{1/2}$ Gagliardo seminorm. We are interested in the minimization of E over a class of maps having finitely many prescribed singularities. We will show that the infimum is equal to the length of a minimal connection relative to a natural geodesic distance D on the plane associated to the matrix field, connecting the singularities. In the case of a continuous matrix field, the asymptotic behavior of minimizing sequences is determined: the energy concentrates near minimizing geodesics for the distance D and this concentration can be described in terms of bubbling-off of circles. This results allow us to compute the relaxation with respect to the a.e. convergence, of the energy functional defined by $F(u) = E(u)$ if u is smooth and $+\infty$ otherwise.

This is a joint work with *Adriano Pisante*.