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Motion by surface diffusion below the roughening temperature

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Abstract

Below the roughening temperature, a corrugated crystalline surface develops facets at its peaks and valleys. The facets grow and merge, producing a uniformly flat surface in finite time. A widely-accepted PDE model for this process is "motion by surface diffusion" (i.e. steepest descent in the H^{-1} norm), using a nonsmooth surface energy with density like $|h_x| + |h_x|^3$. This amounts to a highly nonlinear fourth-order parabolic PDE for the surface height $h(x,t)$. I'll discuss recent joint work with *Irakli Odisharia*, concerning:

(a) a robust numerical scheme for computing the evolution of h; and

(b) an explanation why the evolution is asymptotically self-similar.

The physical correctness of this PDE model remains uncertain. A natural approach to its justification would be to take the continuum limit of a step-flow model. I'll indicate briefly why we do not yet understand this limit.