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Ginzburg–Landau minimizers with prescribed degrees emergence of vortices and existence/nonexistence of the minimizers

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Abstract

Let Ω be a 2D domain with a hole ω. In the domain $A = \Omega \setminus \omega$ consider a class $\mathcal{J}$ of complex valued maps having degrees 1 and 1 on $\partial\Omega$, $\partial\omega$ respectively.

In a joint work with *P. Mironescu* we show that if $\mathrm{cap}(A) \geq \pi$ (subcritical domain), minimizers of the Ginzburg–Landau energy E_κ exist for each κ. They are vortexless and converge in $H^1(A)$ to a minimizing S^1-valued harmonic map as the coherency length κ^{-1} tends to 0. When $\mathrm{cap}(A) < \pi$ (supercritical domain), for large κ, we establish that the minimizing sequences/minimizers develop exactly two vortices—a vortex of degree 1 near $\partial\Omega$ and a vortex of degree -1 near $\partial\Omega$ which rapidly converge to ∂A. In this work it was conjectured that the global minimizers do not exist when $\kappa > \kappa_0$. In a subsequent joint work with D. Golovaty and V. Rybalko this conjecture was proved. The proof is based on an introduction of a related auxiliary linear problem which allows for an explicit energy estimate